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ABSTRACT

The study investigated arithmetic achievement and attitude changes of five institutionalized emotionally disturbed children, age 7 to 11 years. The children participated in a diagnostic teaching program involving diagnosis of mathematics skills and instruction based upon the diagnosis. Emphasized were mathematical patterns, manipulation of physical objects, and problem solving understanding of mathematical actions. A hierarchy of behaviorally defined skills for addition and subtraction was developed. A diagnostic achievement test required Ss to manipulate objects illustrating number sentences, write number sentences describing actions performed on objects by tester, and solve paper and pencil problems. Ss received an average of 38 days of approximately 25-minute instructional periods. At the study's conclusion, both the diagnostic achievement test and a semantic differential attitude scale were administered to the Ss. An educational case study format was adopted as the most feasible method of studying a small number of emotionally disturbed children. At the study's beginning, Bart and David lacked mastery of many tasks concerning addition and subtraction and expressed negative attitudes toward mathematics; both their mathematical skills and attitudes showed improvement. Karen, Cinthia, and Michael also showed skill and attitude improvement. (CB)

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A DIAGNOSTIC AND ACTIVITY BASED ARITHMETIC PROGRAM FOR EMOTIONALLY DISTURBED CHILDREN

U.S.O.E. Grant No. OEG-0-9-470117-3664(721)

by

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1971

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INTRODUCTION

Mathematics[1] can be viewed as a means of developing an aesthetic appreciation for discovering patterns and relationships. Mathematics can also be used to solve everyday problems in the technological society in which we live. Be that as it may, often children do not see the utility of mathematics nor are they able to experience the joy of learning different logical patterns. Perhaps they fear mathematics and view it as a meaningless, drudging exercise. Such may be the case with many children who are classified emotionally disturbed.

Mathematics is sometimes viewed by classroom teachers and their underachieving students as threatening to personal achievement and self-worth. Prime illustrations of this notion would be when a teacher feels insecure in his understanding of various mathematical constructs and his ability to meaningfully explain those ideas to his students in clarifying or skill building situations. The insecure teacher may rely on the use of published and unpublished materials (i.e., worksheets, workbooks, textbooks, programmed materials, etc.) without understanding the mathematical-logical ideas that should be imbedded in the sequencing of such materials. On the other hand, the student who is presented with materials beyond his level of understanding and who then fails at various mathematical tasks will understandably adopt a negative attitude toward mathematics as a helpful tool or as a means of creative expression.

Teachers of emotionally disturbed children often encounter only sporadic success when attempting to provide meaningful mathematics instruction. Such children display learning dysfunctions and respond in aversive and avoidance behaviors to escape learning mathematics in and out of school situations. Success one day may not be enjoyed the next and errors temporarily overcome one week may appear the next. Thus, slow progress often becomes a source of frustration for both the teacher and the learner (Ross, 1969).

Emotionally Disturbed Children

There is a fine distinction between children who are emotionally disturbed and children who experience temporary emotional problems.

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[1] "Mathematics" and "arithmetic" are used interchangeably throughout the study, however, it should be remembered that arithmetic is actually a subset of mathematics.

Redl and Jacobson (1969) emphasized that there is a difference in being disturbing and being disturbed and that there is a difference in being emotional and being emotionally disturbed. Most children, as well as most adults, periodically become emotional and grow experientially from it. However, the emotionally disturbed child and adult become so dominated by emotional problems that behavior becomes an uncontrollable obstacle to learning. Such emotional problems are often long lasting and professional counsel is needed. Therefore, the major difference between being emotional and being emotionally disturbed is the duration and severity of response emotionally displayed (Redl and Jacobson, 1969).

Haring and Phillips (1962) suggested that emotionally disturbed children have serious problems interacting with other people (i.e., peers and authority figures such as parents, teachers, etc.) and for some reason are unhappy about and unable to apply their capabilities commensurate with interests and aptitudes. Such children display a pattern of failure rather than success.

In order to further understand how emotionally disturbed children respond to academic situations, Peter described a students' academic performance that is influenced by emotional problems. He stated:

> Emotional difficulties constitute the greatest number of problems among school children . . . A large number of disturbed children are disabled primarily in their emotional responses. These children suffer from functional disorders which do not involve injury. Their self-concept interferes with their social distortions in their perceptions of their environment (Peter, 1965).

Many emotionally disturbed children are classified as slow learners or underachievers (Haring and Phillips, 1962). Such behavior is reported as being partially due to lack of cooperation, lack of motivation, intellectual retardation, etc.

The Oregon Board of Education (1971) provided broad guidelines regarding a definition of emotionally disturbed children. They stated that:

> Emotionally disturbed children are persons under 21 years of age who require special education in order to obtain the education of which they are capable because they are socially or emotionally maladjusted to the extent that they cannot make satisfactory progress in the regular school program. Socially and emotionally maladjusted children may display problems of personal and social adjustment to other children and to adults in the school or classroom. Their problems are of such intensity as to interfere with efficient school functioning. Their behavior is so deviant and conspicuous that it is readily recognized by the school.

They went on to describe statewide programs and services available to children identified as emotionally or socially maladjusted.

For the purpose of this study, emotionally disturbed children are identified as those children meeting the requirements for placement at Edgefield Lodge Residential Center for Emotionally Disturbed Children at Troutdale, Oregon (1969). These children are described as hyperactive, aggressive, combative, destructive, anti-authority, negativistic, fearful, highly anxious, a school failure, and behaviorally unpredictable. They have few verbal skills, few competencies, little conceptual skill, and a history of failure. Each child in this study was also at least two years retarded in both reading and mathematics, according to nationally normed achievement tests.

Diagnosis and Remediation

A review of the literature failed to provide a large number of references which referred to the teaching of mathematics to emotionally disturbed children. However, various writers (Capps, 1962; Horowitz, 1970; Kolb, 1967; Stone and Rowley, 1964; Tamkin, 1960; Wolkman and Langstaff, 1970) made specific mention of either emotionally disturbed children or socially maladjusted children (delinquent or otherwise) and their difficulty in working with mathematical problems. Each of the above cited authors recommended a carefully designed diagnostic-instructional program to correct specific mathematical deficiencies. An academic program such as they suggested would avoid the introduction of non-essential items; would encourage the presentation of ideas having the greatest amount of carry over value to other situations; and would mean that teachers, or someone close to the teaching situation, have to make decisions regarding mathematics instruction introduced to the children.

Sequencing of Tasks

Several writers (Bidwell, 1969; Briggs, 1968; Bruekner and Bond, 1955; Dunlap, 1971; Gagne, 1967b; Popham and Baker, 1970; Schminke, Maertens, and Arnold, 1971) provided guidelines and made specific suggestions regarding the ordering of behavioral tasks in an hierarchical manner, with specific reference to the sequencing of mathematics instruction. Each of the above cited authors referred to the need for the teacher to determine a specific terminal behavior along with prerequisite tasks that must be mastered prior to the acquisition of that behavior.

In order to determine whether or not a child can perform a given mathematical task with mastery, Gagne (1967b) identified the need to specify hierarchically arranged tasks in behavioral form. Also Gagne (1965) identified at least four major levels of learning that must be

considered when determining the depth of a child's understanding of behavioral tasks. They were: (1) rote or associative learning, (2) conceptual learning, (3) principle learning, and (4) problem solving. Gagne's levels of learning were also hierarchically arranged, from rote learning to the ability to solve problems in a wide variety of circumstances. Gagne found that children who could not grasp prerequisite tasks at a problem solving level of comprehension or performance could not perform higher-level tasks in problem solving situations.

Programming Instruction

Many teachers of children with extreme learning problems have adopted the use of various programmed instructional materials to provide sequentially arranged tasks (Euphemia, 1970). A side-light to the issue of using programmed materials is that the teacher who does not concern himself with the sequential arrangement of tasks must seemingly assume that the authors of the programmed materials have provided the hierarchically arranged tasks within the materials. Also, the teacher who accepts programmed materials at face value must assume that unnecessary items have been deleted.

Seeing the need to involve children in a more active type of learning environment than mere paper-and-pencil programmed materials, Euphemia (1970) provided a rationale for and gave specific suggestions about why children with extreme learning problems should be actively engaged in the learning process. She further recommended that a sequentially arranged task analysis format be used in organizing the instruction. When one task is accomplished then the learner should be guided to the next higher-leveled task. To compliment such a program the use of manipulative and illustrative materials were recommended by Reddell and DeVault (1960), Cotter (1967), and Horowitz (1970). The use of manipulative aids and concrete illustrations was experimentally shown by Piaget (1969) and others as being highly desirable in learning situations for children from birth to approximately eleven-and-one-half years of age. The suggested reason being that children during this period of development need the physical referent in order to think abstractly.

Besides the use of manipulative materials, other suggestions have been made regarding the programming of mathematics instruction for children with learning problems. Dienes (1960) identified a three-step instructional model which he felt should be followed in order for an instructional activity to be meaningful to children. The three steps were: (1) an exploratory period where the learner is exposed to the media and mathematical abstraction to be learned, (2) a period of guided instruction whereby the learner is allowed to discover relationships about the mathematics abstraction under consideration, and (3) a period of appropriate drill and practice to fixate newly acquired mathematics skills. In addition to Dienes instructional model, Harvey (1953)

recommended that a testing, reteaching, retesting format be implemented as a cyclic evaluative way of getting at the specific problems children have in mathematics.

Smith (1969) tied the programming of instruction together with his suggestion that besides the use of paper-and-pencil to determine a child's understanding of mathematics, it is incumbent upon the teacher to allow that child to manipulate objects to demonstrate thought processes and knowledge. To complement Smith's suggestion, Piaget (1952) maintained that in questioning a child regarding his conception of an event, one obtains a reliable and fruitful understanding of how that particular child views that event at that moment in his life. Piaget's interviewing techniques (Furth, 1969) emphasized that the teacher does not give away the desired mature thinking answer at the time of questioning. By gathering information through questioning, teachers may get a closer glimpse of how that child views his world. Then perhaps instruction can be provided to clarify misconceptions and help that child develop skill in needed mathematical ideas.

Model for a Diagnostic and Remedial Program

The investigator looked upon the development of an instructional model as a map or plan. The model for diagnosis and remediation for this study was developed by Gagne. His ideas for a learning hierarchy have many ramifications for the analysis, development, and remediation of arithmetic instruction. Gagne characterized a learning hierarchy as "an ordered set of intellectual skills such that each generates a substantial amount of positive transfer to the learning of a not-previously-acquired higher-order capability." Further, he wrote that "the learning of each entity also requires the recall of relevant verbalizable knowledge, which is not itself represented in the hierarchy (Gagne, 1968).

Some of the most important assumptions supporting Gagne's (1968) ideas about learning hierarchies are:

1. Many mathematical tasks may be analyzed into sets of component tasks, of which different operations are needed to produce them.

2. Subordinate task components are mediators of the final task performance and their absence negates positive transfer such that without them the possibility of a terminal behavior is near zero.

3. The basic principles of instructional design consist of: (a) identifying the component tasks of a final performance, (b) insuring that each of these component tasks is fully achieved, and (c) arranging the total learning situation in a sequence which will insure optimal mediation effects from one component to another. These assumptions are concerned with such things as task analysis, intra-task transfer, component task achievement, and sequencing as important variables in learning.

In order to determine the validity of a hierarchy of tasks, Gagne (1967) suggested that for each task encountered, one simply asks, what would the individual already have to know how to do in order to learn this new capability? Gagne (1967) also suggested two testing methods: (1) check out empirically the individual hypotheses about transfer represented by the hierarchy, and (2) work with the entire hierarchy.

The instructional paradigm proposed by Popham and Baker (1970) was also adopted for this investigation. Popham and Baker made use of Gagne's hierarchy and levels of learning previously described as task analysis, as indicated in Table 1. By specifying desirable behaviors, a pre-assessment of a child's ability to perform those tasks can be made. Once a child's abilities have been assessed, instruction can be provided to help that child achieve previously unacquired skills. By systematically evaluating a child's performance, in terms of his ability to satisfy the criteria established in behavioral objectives, teachers can determine mastery of a specific task before moving on to a higher-leveled task. It would be naive to believe that everyone would agree to all the en route behaviors listed on a hierarchy as prerequisite to a terminal objective. However, the paradigm does allow for the teacher to introduce different approaches leading to the acquisition of needed mathematics skills.

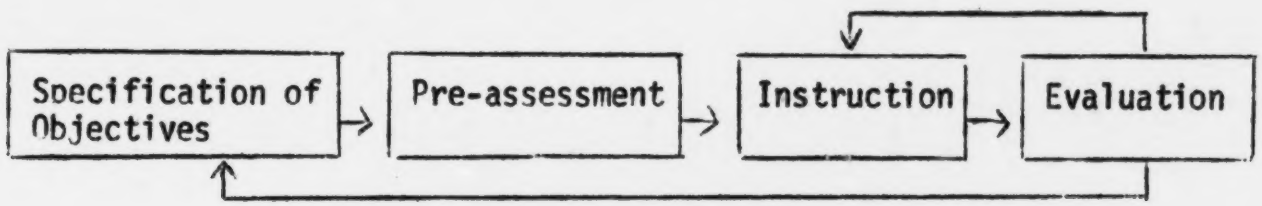
Purpose

The purposes of this investigation were threefold: (1) to develop a diagnostic model comprised of hierarchies of desired tasks, related behavioral objectives with specified criteria, and a related diagnostic test based upon Gagne's levels of learning utilizing paper-and-pencil, manipulative aids, concrete illustrations, and based upon the hierarchies of desired mathematical behaviors, (2) to develop appropriate instructional procedures and subsequent programs for six children specified as emotionally disturbed and (3) to record via case description the results and pertinent information regarding each subject's background and performance prior to and during the time the study was conducted.

Because of the characteristics of emotionally disturbed children and the feasibility of handling only a small number of subjects with careful consideration, it was decided that a case-study format, rather than an experimental design, be employed to gather pertinent information. (See Case-Study Method and Organization of Case-Studies for information regarding methodology.)

Table 1 - Instructional Paradigm

If objectives are not achieved, revise.



If objectives are achieved, augment.

SURVEY OF RELATED LITERATURE

Emotionally Disturbed Children and Arithmetic Achievement

Most of the information written on emotionally disturbed children has been written about modifying deviant social behavior through educational intervention techniques. Little has been written about their abilities to perform arithmetic tasks. In fact, a review of the literature failed to reveal any studies about arithmetic programs designed for emotionally disturbed children. However, several studies were found concerning emotionally disturbed children's attitude toward school and their arithmetic achievement as a component part of academic success.

Maes (1966) reported that underachievement in arithmetic is a characteristic of emotionally disturbed children. He performed a multiple regression analysis on significant variables of a study by Bowers, which analyzed characteristics of a large number of children classified as emotionally disturbed. The variables in decreasing order of prediction were: (1) teacher rating of behavior; (2) arithmetic achievement; (3) intelligence; (4) a class play; (5) teacher rating (physical); and (6) reading achievement. Therefore, according to the data reported by Maes, second only to "teacher rating of behavior," arithmetic underachievement was a common characteristic to the identification of emotionally disturbed children used in Bower's study. This being the case, the arithmetic achievement of emotionally disturbed children is worthy of concern and items related to school achievement need further investigation.

School Achievement

Schorer (1969) analyzed the statements of fifty-two subjects identified as emotionally disturbed children whose ages ranged from five to fourteen years. From the most frequent to the least talked about subjects, the following categories were identified: (1) behavior difficulty; (2) academic failure; (3) academic success; (4) family; (5) teacher; and, (6) feeling about school. Schorer then described each of the categories and concluded that emotionally disturbed children talk about that which is related to school more than any other subject. Also, he concluded that most of the conversation about school was centered around negative aspects of the child's behavior or the behavior of others in the school setting.

Wolf (1965) identified children diagnosed as functionally emotionally disturbed and compared them with their siblings. Although the disturbed group received significantly lower effort grades, no significant differences

in scholastic grades were observed. However, Wolf included only neurotic and personality disordered children and excluded all children who were judged either as slow learners, psychotic, brain damaged, or mentally retarded. The children were matched on a group I. Q. test. It was reported that even though previous research indicated emotionally disturbed children tend to receive depressed scores with group I.Q. tests, the mean intelligence quotient of the disturbed group was higher than their sibling controls.

Schroeder (1965) identified another possible basis for ambiguity on disturbed children's school achievement. She classified children into five categories: (1) those exhibiting aggressive behavior; (3) those having school difficulties; (4) those experiencing school phobia; and (5) those having neurotic-psychotic personalities. Her data showed that educational disabilities were frequently concomitant with emotional disturbance. She also concluded that disturbed children were not a homogeneous group with respect to academic performance. The children having school difficulties had the lowest mean achievement and were found among neurotic and psychotic children. Children displaying behavior in all categories scored consistently lower in arithmetic than in reading.

Motto and Lathan (1966) clarified some of the above mentioned discrepancies on the academic achievement of emotionally disturbed children when they stated that educational improvement had a direct relationship with duration of treatment. They conducted a descriptive study of forty-seven institutionalized emotionally disturbed children in a state psychiatric hospital. The boys in the study had a mean age of 13-1 years and the girls had a mean age of 11-2 years. The information was gathered from the children's cumulative records and from the results of the California Achievement and Arithmetic Tests. Evidence was provided supporting the notion that disturbed children having been at the hospital school ten months or longer benefited more from the educational experience than children who had not. Mental age was used as indicators of improved educational behavior.

Three other studies were reported from which inferences may be drawn about the relationship between emotional disturbance and school achievement. Luszki and Schmuck (1965) reported that elementary school students who perceived their parent's attitude toward school as supportive rather than non-supportive improved in school and maintained greater self-esteem. Brodie and Winterbottom (1967) compared eleven elementary school children, diagnosed as having psychogenic learning disturbances, with a control group of normal children. Clinical interviews with parents indicated that the learning-problem group experiences more derogatory treatment and secrecy needs further investigation. The third study with performance implications was conducted by Graubard (1967), who reported that psycholinguistic correlates of reading disability in disturbed, delinquent children were poorer than those of normal children. This was the case in the discrimination areas of visual motor association, auditory, vocal automatic, visual motor sequences, mazes, and left-right discrimination. He proposed that there were critical visual motor problems in the disturbed children he worked

with and presented suggestions concerning special remedial procedures.

However simplified, Quay, Morse, and Cutler (1966) classified emotionally disturbed children into three general categories. They were: (1) unsocialized, characterized as being defiant, impertinent, uncooperative, irritable and boistrous; (2) unattached, characterized as displaying behaviors of sluggishness, laziness, lack of interest, preoccupation, dislike for school, and inattentiveness; and (3) inadequacy-immaturity, characterized as displaying inferiority feelings, self-consciousness, lack of self-confidence, fearfulness, and depression. They went on to explain that perhaps the displayed behaviors are symptoms of other causative features like: failure in school, problems at home, etc. However, collectively the above mentioned characteristics would have a disastrous effect on academic achievement.

A number of variables have been shown to influence academic achievement; i.e., I.Q., environmental background of the child, milieu, length of stay in an institution, and type of disturbances displayed. Nevertheless, as Quay, Morse, and Cutler (1966) stated, the basic problem remains that children with different patterns of maladajustive behavior may well show different patterns of academic achievement. The tendency of most investigators to use either global definitions of emotionally disturbed children or poorly defined clinical categories is not uncommon. The individual child must be considered as paramount to the suggested treatment.

Arithmetic Achievement and Emotional Disturbances

Several researchers have related underachievement in mathematics to personal adjustment. A study by Capps (1962) investigated the arithmetic underachievement of 188 fourth and sixtn grade students. Students were selected for the study if they fit into the categories of plus or minus one standard deviation from the mean on a standardized arithmetic achievement test. He found that arithmetic achievement tended to be related to personal adjustment. Capps concluded that emotional disturbances tended to foster differences in arithmetic achievement. He also mentioned that children with personal adjustment problems had at least average mental ability. Cleveland and Bosworth (1967) corroborated Capps' conclusions when they found a positive correlation between arithmetic achievement and a healthy personality.

Children classified as emotionally disturbed were found to have lower arithmetic scores than reading scores in two studies, one by Tamkin (1960) and one by Stone and Rowley (1964). However, conflicting evidence was cited regarding such children having an overall educational disability. Tamkin indicated that an analysis of thirty-four residential patients, whose mean age was twelve years, revealed only thirty-two percent having educational disabilities. This was assessed by the reading and arithmetic parts of the Wide Range Achievement Test. However, forty-one percent were identified as educationally advanced and twenty-seven percent were

were performing at grade level. Stone and Rowley found refuting evidence when they studied 116 emotionally disturbed children's responses, whose mean age was nine years. Criticism of Tamkin's study was given regarding the small sample used.

The relationship of delinquency or social maladjustment to arithmetic achievement was investigated in several studies. Feinberg (1947) reported that socially maladjusted boys showed poorest achievement results in the area of arithmetic. Richardson and Saerko (1956) indicated that delinquent prone youths with I. Q.'s below average performed better on non-verbal intelligence tests than verbal intelligence tests. Dinitz, Kay, and Reckless (1957) found that delinquent-prone boys had significantly less arithmetic competence than non-delinquent-prone boys, and an older study by Lane and Witty (1934) found delinquent boys' poorest achievement was in subject areas which required drill, as in arithmetic computation.

Wolkman and Langstaff (1970) reported evidence from published periodical literature that concern for emotionally disturbed children increased over 800 percent from 1963 to 1967. They also cited evidence that the direction for instructional procedures has favored individualizing instruction. Psychoeducational evaluation was emphasized with specific recommendations to pin point specific skills and to develop a hierarchy of behavioral tasks in order to add accuracy for setting appropriate goals for children to attain. The methods for implementing such a program were spelled out in terms of four specified items. They were: (1) A psychoeducational evaluation provides information as to the child's learning pattern; (2) based on the psychoeducational profile of the child, a plan for a comprehensive remedial mathematics sequence is begun; (3) The next step is breaking down the skill levels into specific tasks and defining these tasks in behavioral terms; and (4) The final step in building a remedial program is selecting materials that are applicable and adaptable to implement the teaching of the skills and specific lessons planned.

Horowitz (1970) provided suggestions for teaching mathematics to students with learning disabilities. He identified the following assumptions upon which he made specific instructional recommendations. The assumptions were: (1) The need for a trust relationship for children with psychological blocks in mathematics was emphasized, with additional emphasis on creating an informal instructional environment; (2) Students learn from random and spontaneous experiences; (3) Popular games, hobbies, and related activities can be a basis for instruction; (4) A guide does not have to be a "package" but rather can be an outline of alternatives for experimentation; (5) It is possible to work individually and in small group settings; (6) Memorization alone may hinder the student's ability to apply his knowledge in new situations; (7) Children develop at different rates and age alone cannot be used to determine the level at which a child has advanced; and (8) Instruction should not be initiated with a student unless he feels ready to begin. Horowitz then went on to describe a tutorial program utilizing manipulative materials and provided specific suggestions regarding how manipulative materials could be used in a classroom setting

of children with learning disabilities.

By way of summary, emotionally disturbed children typically have a difficult time being successful in the academic areas of the school curricula and among the most difficult of these subject matter areas is arithmetic. Evidence has been citied to support the notion that most children classified as emotionally disturbed have difficulty learning and applying mathematical ideas. Little evidence is available supporting the use of any of the presently existing programs, except that the sequencing of activities might be a more appropriate way of presenting the material than in an unorganized manner. However, there is also little evidence to support the use of programmed materials in the form of paper-and-pencil. There is evidence supporting the move to provide children having learning disabilities with an individualized approach, pin-pointing specific mathematical problems the learners have, and teaching appropriate corrective skills.

Remedial Programs in Arithmetic

Fenichel concluded an article on psychoeducational approaches for children experiencing serious emotional problems by mentioning that remedial programs should be based upon successful experiences. He said:

> A disturbed child's self-image, like that of any other child, is enhanced by experiencing success and diminished by frequent failure. It is imperative therefore, therapeutically and educationally, that we organize a highly individualized program of teaching and a learning procedure and pace for each child to meet his own capacities and needs . . . As yet there is little solid evidence based on statistical studies or controlled research to measure the value of any special educational program (Fenichel, 1969).

Similarly, Ross (1963) recommended that any remedial instruction should have at least five general characteristics. They were: (1) provide opportunities for the child to be successful; (2) provide immediate feedback of results; (3) provide individual attention and encouragement; (4) use concrete devices for illustrating concepts; and (5) provide adequate drill in a variety of situations. Therefore, a review of remedial programs and/or possible procedures that may be employed to enable emotionally disturbed children to be successful in mathematics was conducted.

Programs Being Used with Emotionally Disturbed

Morse, Cutler, and Fink (1964) made an attempt to gather descriptive data of existing special class programs for emotionally disturbed children. The study indicated that there was confusion on the part of educators about

how to proceed in educating the emotionally disturbed child. They stated that, "There is an obvious need for greater systemization and rigorous research . . . (p. 131)." The work by Cruickshank, et. al. (1961), Haring and Phillips (1965), Karlin and Barger (1969) and Radin, et. al. (1966) provided indications that programming instruction for emotionally disturbed children in a special class setting could be beneficial. Quay (1969) furthered the idea of using programmed instructional materials to develop needed skills and suggested that programmed instruction minimizes the opportunity for wrong answers to occur. In a study of thirty-two first-grade subjects having difficulty with arithmetic, Cotter (1967) identified emotional disturbances as a possible reason for underachievement and recommended individualized instruction through the use of programmed materials to help aleviate the learning problem. She also suggested the establishment of special classes from the beginning of school or an ungraded situation.

Knoblock (1966) in an attempt to improve the education of emotionally disturbed children in regular classes, emphasized consultation services. Balow (1966) after an extensive review of the literature on programs for emotionally disturbed children, reported that the majority of publications have been prescriptions, subjective descriptions, and clinical studies, rather than experimentally based conclusions.

One would assume, by way of summary, that various remedial procedures are presently being used to teach emotionally disturbed children arithmetic. However, little was reported in the literature regarding specific methodology, except the use of programmed materials and an admonishment to individualize instruction. Since there has been little substantiating evidence that any one method has worked better than another in individualizing instruction, further exploration is needed to determine if perhaps a diagnostic-manipulative approach would enable emotionally disturbed children to be successful in mathematics.

Diagnosis of Arithmetic Deficiencies

Current research concerning the diagnosis of specific mathematical deficiencies of emotionally disturbed children is not plentiful. Studies by Buswell (1926) and Eaton (1938) are examples of early attempts to provide teachers with guidelines in diagnosing mathematical learning disorders of children. However, their suggestions are apparently outmoded since the content taught in the classroom has been changed considerably and the emphasis has changed from one of computation to one of understanding underlying processes.

Woodward (1968) recently conducted a study concerned with the diagnosis of mathematical deficiencies of low achieving fifth-grade students. She indicated a need for a hierarchical structure of mathematics when she

found that a child's level of mathematics achievement was a significant factor in understanding related concepts.

A number of procedures used with culturally disadvantaged and educable mentally retarded children perhaps give assistance in developing diagnostic instruments and in preparing subsequent instructional programs. Connelly (1968) and Pritchett (1965) developed diagnostic instruments to be used with EMR children, ages thirteen through sixteen and six through nine, respectively, when determining how children with extreme learning problems develop in relation to normal children. Quick (1967) found that EMR children moved through Piaget's stages of learning, but at a slower pace than normal children. In another study Searle (1968) found that disadvantaged kindergarten children possessed a significantly lower amount of mathematics information than did children from other more advantaged areas and that their mathematical deprivation effected subsequent academic performance. Even though these studies are helpful, they are inadequate as a comprehensive mathematics diagnostic program. Therefore, a review of opinions and research regarding diagnostic techniques was conducted.

Opinions Regarding Diagnostic Teaching

Various writers have identified diagnosis as an intrinsic part of instruction. For instance, Hunter (1966) maintained that diagnosis should be a part of every teaching activity. Implicit within her proposition was that through diagnosing specific needs non-essential tasks can be eliminated, or at least the teacher may be aware that they are non-essential. Accompanying the above mentioned suggestions, Hunter recommended the following four questions be considered in a teaching-learning act: (1) What objective is appropriate for this learner? (2) What is the learner's present status in relation to that objective? (3) What is the next learning step in the attainment of that objective? and (4) What can the teacher do to help the child make that step efficiently and economically?

Gagne (1965) defined learning and then provided recommendations regarding the structuring of learning and diagnosis of specific deficiencies. Gagne defined learning as ". . . a change in human disposition or capability which can be retained and which is not simply ascribable to the process of growth." Therefore, according to Gagne, learning occurs sometime prior to the time a given behavior is displayed. When learning occurs, the individual is able to accomplish something that he could not accomplish before being presented with a learning situation.

Gagne identified eight types of learning after he defined learning. They were: (1) signal learning, (2) stimulus-response learning, (3) chaining, (4) verbal association, (5) multiple discrimination, (6) concept learning, (7) principle learning, and (8) problem solving ability. Each type of learning identified conditions under which changes in capabilities of human learning are brought about. The types of learning suggest a hierarchy. If problem solving is the desired learner behavior then prerequisite

principles must be applied to the learning situation. If principles are to be learned subordinate concepts must be learned previously. Also, if concepts are to be learned the factual data or rote associations must be part of the experiences of the learner. Thus, it becomes theoretically possible to work backward from any given objective to determine what prerequisite learnings must be displayed. Therefore, the most readily apparent implication utilizing the types of learning identified by Gagne and modified by the investigator of this study is directed to the diagnosis of needed skills and subsequent instruction.

Hewett (1967) developed and used an engineered classroom design based upon the behavior modification model in both institutional and public schools. The basic premise for his model was that there is a hierarchical arrangement of behaviors which leads to the acquisition of a socialized or academically oriented desired goal and it is the teacher's responsibility to realize the level at which the learner is behaving and provide remediation at that level of performance. Hewett, Taylor, and Artuso (1968) reported that children in the engineered classrooms maintained a 5-20 percent task-attention advantage over children in the control classrooms. Gains in arithmetic fundamentals were significantly correlated with the use of the engineered design, but reading and spelling gains were not significantly different between the experimental and control conditions. Hewett's work has special significance to this study since the methods were developed specifically for emotionally disturbed children.

Goodlad (1966) advocated creative diagnosis and prescriptive teaching as being necessary to eliminate teacher talk and promote individual instruction. He further provided four suggestions regarding the implementation of the above mentioned attributes. They were: (1) develop systems for collecting data necessary for diagnosis; (2) teachers must become adept at interpreting data; (3) curriculum guides must be developed by specialists using taxonomical analysis of behaviors and concepts; and (4) pedagogy with a wide range of instructional alternatives should be utilized.

Other recommendations such as sequentially arranging questions and sequencing behavioral goals have been made. For instance, Suchman (1967) suggested diagnostic teaching because it gets directly to what the learner knows and accepts. Also, Suchman stressed the sequential arrangement of questions, thus being aimed at the development of a learner awareness of the cognitive structure implicit in the material to be learned. Ponder (1968) recommended the ordering of learning goals into manageable steps which may be translated into behavioral terms. Utsey (1965) expanded on Ponder's suggestions by recommending the acceptance of children at their present level of performance, accompanied by continuous assessment and employing the use of informal diagnosis of skills, attitudes, work habits, etc. By becoming a learning diagnostician, Eisele (1967) suggested teachers may determine the needs of individual learners. As a diagnostician, teachers perhaps can overcome many uneasy feelings about not knowing where to begin instruction.

Brueckner and Bond (1955) provided a discussion of "techniques of diagnosis." The topics considered were: (1) nature of diagnosis; (2) approach to diagnosis; (3) levels of diagnosis which included general diagnosis, analytic diagnosis and case-study procedures; (4) techniques of analytic diagnosis; and (5) case-study methods. That which was referred to as "nature of diagnosis" was viewed as essential in classroom instruction. Brueckner and Bond cited competencies essential for skill in diagnosis: (1) thorough understanding of the learning process in the area being investigated; (2) knowledge of factors that contribute to learning difficulties; (3) skill in considering more than one hypothesis regarding causation; (4) ability to utilize diagnostic methods and devices; (5) ability to realize when more sophisticated professional help is needed; (6) ability to synthesize information from conglomerate sources; and, (7) ability to recommend corrective procedures and demonstrate the methods and materials of instruction to be used.

The "approach to diagnosis" suggested by Brueckner and Bond incorporated the following seven steps: (1) setting up educational objectives; (2) appraising present achievement of accompanying behavioral competencies in order to determine strengths and weaknesses; (3) typing together previous experiences with what research has stated regarding growth and progress; (4) preliminary surveying of the case under investigation to determine factors that should be systematically investigated; (5) systematic testing and analyzing the component elements of the area of learning involved to determine the extensiveness and seriousness of the disability; (6) planning of procedures for improvement; and, (7) determining the validity of the diagnosis and the effectiveness of the corrective measures taken.

In order to implement the "approach to diagnosis" Brueckner and Bond suggested the use of three levels of diagnosis: (1) general diagnosis; (2) analytical or differential diagnosis; and, (3) case-study procedures. That which was meant by "general diagnosis" was an overview of capabilities. Perhaps, as mentioned before, achievement tests fall into this category.

"Analytic diagnosis" provided the basis for what is meant as a diagnostic approach to teaching. That which was meant as analytic diagnosis involved the systematic procedures for locating and identifying specific weaknesses or shortcomings in the curricular area under consideration. With specific reference to diagnostic teaching, Brueckner and Bond recommended case-study procedures or the application of clinical diagnostic techniques as an effective data collection procedure. Case-study procedures enable the teacher to study in detail the performance of an individual student, having evident learning problems. Brueckner and Bond did not go into detail regarding case-study procedures other than perhaps the suggestions made earlier regarding what teachers might look for in a diagnostic approach to teaching was to cope with the nature and seriousness of learning difficulties and rectify misconceptions by providing instruction to correct deficiencies.

To summarize the opinions of the above mentioned writers and

theoreticians, Wayson (1967) maintained that diagnostic teaching would result in vastly improved student learning productivity. Wayson also concluded that a diagnostic teaching program would require ability and incentive to develop acute observational and instructional techniques.

Use of Programmed Instruction

Bierden (1970) found success using a mathematics program emphasizing an intra-class grouping plan. Group instruction, followed by independent work with individualized objectives, resulted in significant gains in computational skills, concept knowledge, and attitude, with a reduction in anxiety.

Two programs make special provision for instructional organization. In neither case was there evidence to show that the special provision makes for improved achievement. Individually Prescribed Instruction (IPI) and the Program for Learning in Accordance with Needs (PLAN) both (1) involve the use of behavioral objectives; (2) identify activities and materials to meet those objectives; and, (3) provide sets of test items for those objectives. However, both programs do not incorporate manipulative models for diagnosis or instruction, but are basically paper-and-pencil oriented. Earlier research indicated that achievement results with IPI were not, in general, different from those with conventional elementary school mathematics programs (Suydam and Riedesel, 1969). Recently, Ferney (1970) reported that fifth graders not using PLAN achieved significantly higher on arithmetic reasoning than the group using PLAN. Girls using PLAN achieved higher scores than did boys, and thus PLAN may be more appropriate to the learning styles of girls.

Scott (1969) matched twenty-five pairs of low-achieving seventh-graders on computation, concepts, and applications. One-half then used programmed materials appropriate to meet diagnosed needs. They made significantly greater gain scores in computation than did students in the regular classroom, although differences on concepts and applications were not significant.

Use of Diagnostic Instruments

Gronlund (1969) identified several reservations that should be kept in mind when selecting and using diagnostic tests. Among them were: (1) in selecting a test, the diagnostician needs to evaluate the test in light of desired information; (2) diagnostic tests indicate typical errors, but do not indicate underlying causes; and, (3) diagnostic tests tend to have low reliability because of relatively few items measuring each type of error.

Ashlock (1967) indicated his concern for standardized diagnostic tests not yielding precise information. He further explained that teachers

need to identify the thought processes and skills that are prerequisite to behavioral tasks. He also stressed that commercial diagnostic tests are not the best way to diagnose learning difficulties since they may or may not include items for each prerequisite understanding or skill necessary in order to perform various mathematical tasks. Thus, teachers may find it necessary to construct their own diagnostic tests.

Similarly, Burns (1965) and Ross (1963) indicated the need to use diagnostic tests to pin point the particular skills that a learner does not possess. Burns indicated that teachers should purchase or construct tests for diagnostic procedures which contain problems corresponding to particular skills or concepts. Furthering this notion, Ross suggested that teachers should construct informal arithmetic inventories (IAI). When constructing informal tests, Ross indicated that the diagnostician needs to look for specific behaviors, thus making it necessary for him to know the underlying mathematical logic behind the tasks that he asks the learner to perform.

Earlier studies by Grossnickle (1935) and Brueckner and Elwell (1932) were concerned with reliability of diagnostic instruments. Grossnickle conducted a study to determine the source of errors using a written division diagnostic test. He used 200 fifth to twelfth grade students as subjects and gave them the written test. He found that pupils needed at least three responses to each fact before diagnosis of singled out mathematical understandings could be reliably determined. Using a similar technique, Brueckner and Elwell provided corroborative evidence regarding the use of three chances and explained that it was necessary to alleviate the chance of error.

Utilizing a diagnostic-manipulative technique with underachieving fourth graders, Dunlap (1971) conducted a study which compared two experimental arithmetic treatments; the Laboratory Treatment and the Textbook Treatment. The Laboratory Treatment group, comprised of 74 subjects, were provided with instruction which emphasized manipulating objects, writing appropriate number sentences describing mathematical actions performed, and solving problems with paper-and-pencil. The Textbook Treatment group, comprised of 73 subjects, received instruction emphasizing a page-by-page presentation of commonly used textbooks. The subjects were randomly assigned to treatment groups. All subjects were taught on a small group basis. After twenty-nine days of instruction, mathematics achievement was measured for both treatment groups according to computational skills, knowledge of concepts, and application of processes. Using a standardized diagnostic test Dunlap found that there were no significant differences in achievement. However, using an experimenter designed diagnostic test, based upon a mathematical hierarchy and emphasizing the indications of achievement mentioned above, he found a significant difference favoring the Laboratory group regarding understanding process ($p < .05$) and a significant difference favoring the Textbook group regarding computational skills ($p < .01$). Attitudinal differences with respect to arithmetic was negligible.

Summarizing the evidence regarding diagnosis of arithmetic skills, it is possible to see that zeroing in on specific mathematical difficulties children are having can be helpful. Also, providing an instructional format based on the diagnosed findings perhaps is an answer to working with emotionally disturbed children. The level of understanding should not be neglected, implying that if a child understands a mathematical operation or process then he will be able to apply that knowledge in problem solving situations with more than mere rote or association understanding. Also, implicit in many of the writings regarding the teacher as a diagnostician is the idea that manipulation of objects can perhaps play an important role in building a deeper understanding of a mathematical process.

Learning and Use of Manipulative Aids

Kieren (1969) summarized active learning as a basis for meaningful understanding in arithmetic. In summarizing a review of the literature available to him, Kieren concluded that the literature presented at that time could be characterized as "theoretical discussions or descriptions of developmental efforts," and mentioned that many of the studies cited were only pilot projects. Part of that which Kieren described as active learning involved the role physical materials play in helping children develop more firm mathematical understandings of relationships between themselves and the physical world in which they live.

Theory of Learning Followed in the Study

Bruner (1964) postulated that mathematical ideas develop in three steps: (1) enactive; (2) iconic or image manipulative; and, (3) symbolic or abstract thinking. The enactive step requires the use of physical materials as a referent since a child functioning at this level does not possess a mental image of the mathematical operation under consideration. A child in an iconic level has an image of the physical object and understands the mathematical operation and its relationship to that object. The action of the mind can be mentally manipulated as long as there is a referent. However, Bruner's most mature level of learning would be the symbolic step, whereby the learner theoretically has little need to have the physical referent present. Bruner indicated a danger in using instruction in isolation from the real object. If a learning or problem solving symbolic transformation fails a person, Bruner suggested that he needs to have the appropriate physical referent or iconic transformation available to him. Bruner went on to explain that secondary school children often function almost entirely at the symbolic level, but periodically need the iconic transformation in order to solve their mathematics problems. Bruner mentioned that mature cognitive knowing develops in the sequence described and made reference to the work of Piaget for support

of developmental learning.

Piaget (Phillips, 1969) mentioned that beginning at birth all organisms, man included, assimilate or take in information and accommodate their lives to that newly acquired information. The process of assimilating and accommodating he called adaptation. He also mentioned that an organism is most often in a state of equilibration since the way he views his world is viewed logically for that moment in his life. Assuming this process to be true, one would conclude that most, if not all, behavior is a product of that which he has learned. However, Piaget also emphasized the role maturation plays in development, with learned behavior and maturational growth complementing actual performance. By way of implication, teachers' tasks consist of providing a rich environment, identifying sources of misconception and then providing information and experiences based upon that which the child cannot understand or perform.

Accompanying the principle of adaptation is the notion that each human organism develops in a predictable and sequential manner. For instance, Piaget (1969, 1961) identified and substantiated the existence of four major developmental periods, which maturationally influence how children view their world. They are as follows:

1. The period of sensori-motor intelligence begins at birth and continues to approximately two years of age. This period occurs before the advent of language and results in performances such as the organization of spatial relationships, and the organization of causal relationships.

2. The period of pre-operational thought occurs at approximately two years of age to seven or eight years of age. Pre-operational thinking begins with the symbolic or semiotic function. This period is characterized by children displaying representational thought by means of the manipulation of symbols. Though the pre-operational child cannot perform operations, which are internalized acts that are reversible, they do perform acts leading in opposite directions of thought processes. Pre-operational children develop classification schemes based upon logical structures necessary as organizers for operational thought; however, they have difficulty in seeing the world from another person's point of view. Also during this period children often gain a command of language.

3. The concrete operational thought period begins around the age of seven or eight years and is characterized by the inception of operations. Initially the operations are concrete; that is, they are used directly on objects in order to manipulate them. During this period children can classify concrete objects, order them, establish correspondences between them, use numerical operations on them, and measure them from a spatial point of view. However, operations remain concrete

until children are eleven or twelve years of age.

4. The period of formal operations or propositional thought begins when a child is eleven or twelve years of age. At approximately this age the operations are no longer applied solely to the manipulation of concrete objects. Children during this period are capable of proposing viable hypotheses about abstractions and can reach deductions by formal and logical means.

Ability to view one's world in a mature formal operations manner is often reached around the age of fifteen years. However, in light of this study and with full recognition of individual development of children, Piaget made the following statement:

> ". . . these statements have essentially a biological meaning, in the sense that the order of the stages is constant and sequential. Each stage is necessary for the following one. If this were not the case, one would be in no position to talk of stages. Naturally the age at which different children reach the stages are accelerated, whereas in others they are more or less systematically retarded. This differential development shows that stages are not purely a question of maturation of the nervous system but are dependent upon interaction with the social environment and with experience in general. The order, however, remains constant (Piaget, 1969).

Support for the Use of Manipulative Aids

Educators support the use of manipulative objects as models for teaching arithmetic to young children. For instance, Drasin (1957) proposed the use of a three-step procedure in mathematics instruction. The three steps were: (1) concrete, (2) semi-concrete, and (3) abstract. She further explained that the semi-concrete step is to bridge the understanding of the concrete with the understanding of an abstract notion. She also mentioned that the use of manipulative models help children to concentrate and strengthen their understanding of process.

Isaacs (1964) cited the use of environment as a natural stimulus for teaching mathematics. He suggested the need of having a great variety of materials to enrich the environments for children coming from sterile environments. Davis (1967) maintained that physical materials added reality to the teaching-learning situation and provided an alternative to authoritative, computation-oriented teaching. Dienes (1967) advocated the use of concrete materials and games by stating that they foster concept development. He cited children's needs to identify perceptual and mathematical variates of a concept through repeated exposure to differing circumstances. E. Biggs and MacLean (1969) supported the use of activity learning by proposing a laboratory approach to learning, as recommended

by proponents of the Nuffield Mathematics Teaching Project. In a related way, J. Biggs (1965) claimed that a multi-model environment for mathematics was superior to a unimodel environment, which uses only one type of manipulative material such as Cuisenaire rods.

Sutton-Smith (1967) discussed research on the role of play in cognitive development. His research supported the use of play as a means of increasing a child's response repertoire. Dienes (1964) supported Sutton-Smith by emphasizing the role of elemental play and mathematical rule-based games in the elementary school. Dienes and Sutton-Smith maintained that by using games the student can cope with advanced mathematical ideas and such learning correlates highly with information-seeking behavior.

Games are often included when teaching arithmetic skills. In a study by Bowen (1970), using fourth graders as subjects, it was reported that pupils who used logic games had significantly higher gain scores than those who used a textbook when studying mathematical logic. Wynroth (1970) reported that kindergarteners and first graders, taught new concepts verbally through a series of competitive games, followed by self-paced written work later, had significantly higher scores on achievement tests than those who had a "normal" program. Wynroth noted that when games are included in the program they should be carefully selected, supervised, and used to meet specific objectives.

Sanders (1964) suggested four ideas that should be remembered regarding the use of physical objects as models. They were: (1) each child should use the model at his desk to solve his problems; (2) models must be an actual application of the mathematical principle presented; (3) all prerequisite skills and concepts must be presented prior to use of the models; and, (4) each child must know how to use the model.

Complementing Sanders, Bernstein (1963) indicated that there should be a correlation between the operations which are carried on with a manipulative device and the operations which are performed when doing the same mathematics idea with paper-and-pencil. Also, manipulative aids should involve moving parts which illustrate to the learner the process involved in a mathematical principle. Bernstein also accentuated the use of manipulative and pictorial materials which emphasize the use of as many senses as possible.

Osborn (1961) and Hertz (1957) also provided guidelines for the use of manipulative aids. Osborn emphasized that models should resemble the actual physical situation and be clearly understood by the learner so that valid conclusions can be drawn. Hertz identified three ways manipulative aids should be used in arithmetic. They were: (1) to help the teacher demonstrate the meaning of a new concept; (2) to help the learner as he practices for ease in computation; and, (3) to help the learner as he practices to gain speed and accuracy in recall. Hertz went on to mention that more than one device should be used to introduce a new concept to help children understand that the mathematical idea is generalizable to

more than one situation.

In separate studies Williams (1967) and J. Biggs (1965) reported the results of three year studies in England and Wales, respectively. Three structured methods of instruction were compared to traditional methods of teaching. The three experimental methods were: (1) Cuisenaire Rods (unimodel); (2) Stern materials (unimodel); and, (3) Dienes materials (multimodel method). The experiments either compared students in separate schools or compared students with previous students in the school who used traditional methods. Both researchers tried to control for student ability, teacher experience, constancy of method, and the Hawthorne effect. Biggs found that high IQ boys fared better on understanding, motivation, and attitude under unimodel methods than under traditional methods. The traditional methods appeared to produce good computational facility but created high anxiety. The multimodel group showed superior results to the unimodel and traditional method groups on the three criteria mentioned above.

Swick (1959) conducted a research study using multi-sensory manipulative aids with 404 second and third grade subjects, approximately half of which were put into a control group without the use of the manipulative aids. He concluded that multi-sensory aids did significantly change the subject's computational and reasoning abilities.

Harshman (1962) conducted a study involving three treatment groups. They were: (1) expensive materials (Numberaid Program); (2) inexpensive multiple manipulative materials; and, (3) low cost materials (less than $.95 per child). The study involved 654 first grade students. He concluded that the subject's reasoning or computational abilities were not significantly different as measured on the Stanford Arithmetic Test Primary Battery, Form J after five months of instruction. He recommended that the following items needed closer control in all treatment groups: (1) pace of presentation; (2) selection of advanced content; and, (3) methods of using material.

Davison (1968) provided an extensive list of commercially prepared manipulative materials. Each item was labeled with accompanying source of purchase, proposed retail price, and brief physical description. Each item was classified and listed under one of the following fifteen categories: Blocks, Calculators/Computers, Cards, Construction, Drawing Tools, Geoboards, Measuring Devices, Miscellaneous Items, Models, Numerical Games, Puzzles, Shapes and Tiles, Strategy Games, Student Instructional Materials, and Teacher Resource Materials. The list was not intended to be exhaustive. One noted item is, however, that there is not any noncommerical items described. The Arithmetic Teacher, Instructor, Grade Teacher and other periodicals abundantly identify possible teacher and student-made devices to use. There is no evidence to support the notion that commercially prepared materials are more effective than non-commercial materials.

Effect of Attitudes on Arithmetic Achievement

Wilson (1961) maintained that before "progressive education" more school failures were caused by arithmetic than by any other subject. Little evidence is available to suggest the modern curricula have fostered more positive attitudes toward arithmetic. A committee formed to explore the issue concluded that more information was needed about biological inheritances and home backgrounds of the pupils, attitudes and training of teachers, and the content organization, goals, and adaptability of the curricula (1956). In order to view how attitudes influence children's arithmetic achievement, what seems necessary is a review of literature regarding the following subheadings: (1) Methods of Measuring Attitudes Toward Mathematics; (2) Formation of Attitudes Toward Arithmetic; (3) Instructional Procedures and Attitudes; and, (4) Relationship Between Attitude and Arithmetic Achievement.

Methods of Measuring Attitudes Toward Mathematics

A variety of techniques have been used to measure attitudes of learners toward arithmetic. Corcoran and Gibb (1961) described three general techniques: (1) observational methods; (2) interviews; and, (3) self-reporting methods such as questionnaires, attitude scales, sentence completion, projective techniques, and content analysis of essays.

Observable behavior has been used as criteria for attitude assessment. Brown and Abel (1965) found that observations conducted by teachers were inadequate when appraising their students' attitudes toward mathematics. However, in a study of high school students, Ellingson (1962) reported a possitive correlation of $r = .48$ between inventoried attitudes of 755 students with their teacher's ratings of their attitudes toward mathematics. Also, in an experimental study involving a semi-structured interview technique, Shapiro (1962) utilized nineteen questions designed to determine the feelings of fifteen boys and fifteen girls. Three judges were used to analyze the interviews. Shapiro found a positive correlation between the attitudes of the subjects toward arithmetic and scores on the California Achievement Test.

Questionnaires have been used to determine attitudes about mathematics. Dreger and Aiken (1957) administered three questionnaire items with expected true-false responses and Kane (1968) constructed another questionnaire to measure attitudes toward mathematics and other school subjects. However, the most common questionnaire type of technique is the attitudinal scales.

Attitudinal scales come in at least five different forms. One is the self-report measure involving the rank ordering of positional responses. Allport and Hartman (1925) initiated and Thurstone and Chave (1929) modified this approach by providing "equal appearing intervals"

with check off items from which the respondent chooses his answer. An example of a Thurstone scale of attitudes toward arithmetic is Dutton's scale (1962) where fifteen items, consisting of positive and negative attitudes toward arithmetic were presented. A second method of summed ratings was introduced by Likert (1932). According to Scott (1968), the respondent is asked to indicate the degree of agreement or approval to all items on a five-point scale. Various researchers (Gladstone, Deal, and Drevdahl, 1960; Aiken, 1963; Adams and Von Brock, 1967; Dutton and Blum, 1968) have preferred Likert Scales because of their comparative easiness to construct and because these scales can be specifically designed to assess attitudes toward mathematics. In 1950 Guttman introduced a third attitudinal scale, the scalogram analysis, which involves dichotomous data like "yes-no" and "agree-disagree" responses. An example of a Guttman scale was utilized in a longitudinal study conducted by Anttonen (1968. He arranged ninety-four attitude scale items into fifteen Guttman-type scales. Fifth and sixth grade students and the same students when in the eleventh and twelfth grades were subjects in the study. He found the following results:

a. The correlation between students attitudes toward math from the fifth and sixth grades carry over to the eleventh and twelfth grades ($r = .305$, $p < .01$).

b. There was a low correlation between elementary attitudes scores and quantitative thinking when compared with secondary attitude scores and mathematics GPA.

c. Quantitative thinking and mathematics GPA correlated highly ($r = .804$, $p < .01$).

d. For both grades and sexes there was a low positive correlation ($p < .01$) between elementary and secondary mathematics attitude scores.

e. For twelfth grade females the correlation between elementary attitude scores and quantitative thinking was significant at $p < .05$. Anttonen concluded that something must be done to change the "emotional blocks" or "mathophobia".

In 1965 Coombs introduced an unfolding technique involving a series of items where the respondent was asked to check off the one which best fit his position on an issue. A fifth type of scale was a self-reporting technique which is not dependent upon the respondents' opinion statements. It was utilized by Maertens (1968) in a study with third grade subjects. The instrument is called a semantic differential and utilizes a seven point continuum, offset by eleven adjectives, which the respondent is asked to indicate his attitude toward mathematics.

Other methods have been used to measure attitudes toward arithmetic. Nealeigh (1968) utilized a picture-preference test as a projective measure

of pupil attitudes and "achievement proneness" in mathematics. The students were individually shown 310 pairs of pictures (one member of each pair contained a math concept) and the respondent was told to indicate which of the two pictures he preferred. The "math concepts" included in the pictures were those of symmetry, similarity, order, and pattern. Attitude and achievement in mathematics were assessed in another way and compared with pupil responses to the picture preference test. Although certain pictures discriminated between pupils with positive and negative attitudes and between pupils with high and low achievement, the pictures that were chosen by third graders were not necessarily the same ones chosen by seventh graders. Dreger and Aiken (1957) measured changes in electrical skin resistance (GSR) in 40 college students while the Verbal scale of the Wechsler-Bellevue Intelligence Scale (WB-I) was being administered. They found subjects independently identified as anxious about mathematics reacted significantly different on the GSR when the arithmetic portions of WB-I was presented than did subjects who were not highly anxious about mathematics. Also, Milliken and Spilka (1962) measured breathing depth, breathing rate, blood pressure, heart rate, and GSR during the first and last thirty seconds of the time their subjects were taking each subtest of the American Council of Education Psychological Examination (ACE). The results showed that subjects who were low in mathematics and high in verbal scores on the Scholastic Aptitude Test gave greater physiological responses during the administration of the ACE quantitative tests. In addition, males in general gave greater physiological responses during the ACE linguistic tests than during the quantitative tests; the reverse was true for females.

Formation of Attitudes Toward Arithmetic

Morrisett and Winsonhaler (1965) suggested it is generally recognized that attitudes toward mathematics in adults can be traced to childhood. Fedon (1958) and Stright (1960) provided evidence that definite attitudes toward arithmetic may be formed as early as the third grade, but these attitudes tend to be more positive than negative in elementary school. For instance, a survey by Herman (1963) of the subjects least preferred by a group of fourth, fifth, and sixth graders found that arithmetic was typically in the middle when subjects were ranked from least to most preferred. For boys, the order of the five subjects from least-liked to most-liked was English, social studies, arithmetic, science, and spelling. For girls, the order from least-liked to most-liked subjects was social studies, science, arithmetic, English, and spelling. However, a study involving 43,979 fifth grade students by Curry (1963) in answering a questionnaire regarding subject preference, the order of art, arithmetic, spelling, and reading as the number one through four subject preferences was indicated.

Numerous studies contradict the notion that most students dislike mathematics. Studies by Inskeep and Rowland (1965) and Mosher (1952)

indicated that students frequently selected arithmetic as their favorite subject. Many authors (Anderson, 1958; Chase, 1949; Herman, 1963, Greenblatt, 1962) reported that arithmetic as a preferred subject was above average. Still other authors (Chase, 1949; Curry, 1963; Dutton, 1956) found middle grade boys rating their liking for arithmetic slightly higher than girls; but Stright, when including lower grades, found girls showing slightly higher preference. Chase's study indicated that geographical location of children made a difference in subject preference. This was also substantiated by Mosher (1952).

The influence teacher's attitudes toward mathematics have on their students cannot be answered directly since various studies have reported differing results. Inskeep and Rowland found no relationship between teacher and pupil attitudes, but Chase found pupils of teachers who preferred arithmetic appeared to favor it themselves. With high-intelligence pupils Greenblatt reported the preference of teachers corresponded highly with that of their students.

Instructional Procedures and Attitudes

Certain authors (Lyda and Morse, 1963; Malone and Freel, 1954) reached conclusions to support the statement that when arithmetic is taught as a skill having practical value in out-of-class situations, attitudes become more positive. They also found that higher sustained attention was achieved when students would associate a practical need with a mathematical idea. Fedon (1958) found a positive increase in attitude when problem solving was related to experiments. Both presentation of arithmetic by television and specific review seemed to create positive attitudes, as reported by Kaprelian (1961) and Burns (1960), respectively. Dutton (1956) found that students report lack of understanding, difficulty, poor achievement, and boring aspects of arithmetic as major reasons for their dislike of arithmetic.

Relationship of Arithmetic Achievement and Attitude

Research by a number of authors (Bassham, Murphy, and Murphy, 1964; Dean, 1950; Powell, O'Connor, and Parsley, 1964) indicated that there is a positive relationship between attitude and achievement in mathematics. However, they also indicated that it was difficult to measure precisely the relationship since there are numerous interacting variables. Anttonen (1968) found there was not a significant relationship between attitude and achievement. Greenblatt found girls with high arithmetic achievement had positive attitudes toward math. Rowland and Inskeep (1963) found a feeling of success increased preference and attitudes toward arithmetic. Rice (1963), Greenblatt (1962), and Stephens (1960) found that gifted or accelerated pupils had a higher interest in arithmetic than did other children. A study having related findings was one by Feldhusen and Klausmeier (1962) where they found a significant

relationship between anxiety and low arithmetic achievement for low I.Q. pupils.

Research has indicated a difference of opinion regarding the relationship attitudes have on arithmetic achievement. Perhaps part of the reason is the lack of uniformity of instruments used to gather pertinent data regarding attitudes and achievement or perhaps it is difficult to control the number of interacting variables. Whatever the case, indications are that researchers are not completely satisfied with presently existing attitude indicators. However, as Purkey (1970) reported, the evidence clearly shows a persistent and significant relationship between self concept or attitude and academic success. Relating this generalization to this study and as corroborated by Radke-Yarrow, Davis, and Trager (1949), there is also a connection between self concept and behavior.

Purkey (1970) summarized that which would be indicative of students' attitudes toward mathematics instruction as follows:

> What those studies seem to indicate is that the image of school grows gradually less positive with time, and communicates a sense of personal inadequacy to many students. If we are serious about making an understanding of self concept a central part of the school, then we must seek out ways of modifying our educational methods to prevent the development of negative self concepts in students. Teachers need to give serious application to learning the principles which call for a careful evaluation of present school policies in the light of what we know about the self and scholastic success. Once a child becomes convinced that school is not the place for him, that is a place of threat and anxiety where he cannot hope to succeed and where his identity is lost, then the school as well as the student is in very difficult position.

PROCEDURES

This study was designed to determine the effects of a diagnostic-manipulative approach to teaching arithmetic to six institutionalized emotionally disturbed children. The ages of the children selected for the study ranged from seven to eleven years. At the onset of the study it was recognized there were a large number of uncontrollable variables (i.e., lack of randomized selection of subjects, unpredictable behavior of the children of which the selected sample was a part, lack of previously tested diagnostic instruments identifying hierarchically arranged observable behaviors which emphasize the use of manipulative materials, lack of selectivity and equivalency of teachers' backgrounds, inability to generalize to larger population samples because of problems unique to individual subjects, and short attention span of the subject selected). Therefore, a case-study method was adopted.

Three steps were followed in the study: (1) a diagnostic model for teaching addition and subtraction of whole numbers was adopted, with accompanying researcher-designed mathematical hierarchy, related behavioral objectives, and diagnostic test. The diagnostic test was based upon prerequisite tasks necessary to be mastered before a terminal objective could be accomplished with mastery; (2) an instructional procedure emphasizing the need for the use of paper-and-pencil and manipulative materials was utilized; also, the instructional procedure emphasized the understanding of process rather than mere rote or associative learning; and (3) a case-study data collecting model was developed to gather pertinent information.

Setting of the Study

Selection of the School

Redl and Jacobson (1969) are among the writers who maintain it is difficult to distinguish emotionally disturbed children from children with temporary emotional problems. With this in mind, it became readily apparent to the investigator that a sample of children clinically identified as emotionally disturbed was difficult to obtain in a large enough sample to conduct an organized experimental study. Therefore, the investigators looked for a centrally located sample of emotionally disturbed children with which to conduct the investigation. The administrative and teaching staff at Edgefield Lodge Residential Center for Emotionally Disturbed Children at Troutdale, Oregon were approached and accepted the challenge to work cooperatively in the study.

Description of Edgefield Lodge

Edgefield Lodge is a special school for emotionally disturbed children. The school is located about fifteen miles east of Portland, Oregon city center. It is a residential rehabilitative center for up to thirty boys and girls, ages six to twelve, where they and their families can obtain therapeutic assistance to modify anti-social behaviors in the children. At the time this study was conducted, Edgefield Lodge was governed by an advisory board, under the jurisdiction of the University of Oregon Medical School. The director of Edgefield Lodge was Dr. Buell Goocher. Mrs. Floy Pepper was the Coordinator of Instruction for Multnomah County Education District and functioned as the principal of the school. Edgefield Lodge is located on a five acre rural campus, having been converted from a TB hospital, and consists of a one-story brick building and two auxiliary buildings containing the arts and crafts and workshop activities. The school is supported by tax revenue, half from Multnomah County and half from the State Mental Health Division. The school admitted its first five children in March 1965 and was at full capacity of thirty children from February to July 1970 when the data for the study was collected. Administratively the school functions under the Department of Medical Services. The director of the school is accountable to Public Health Services.

The educational orientation of the staff at Edgefield Lodge consists of three mutually dependent functions. They are: (1) treatment, (2) training, and (3) research. The school emphasized behavior modification techniques; i.e., identifying specific behaviors and changing maladaptive behaviors to more acceptable ones using intervention reinforcement. This study was not concerned with the deviant behaviors per se. Since the school's emphasis was also to provide the children with successful academic experiences, this study identified selected children's success or non-success in learning certain arithmetic skills.

Teachers in the Study

Two of the five certified teachers at Edgefield Lodge were cooperatively selected by the investigators and the principal of the school. Both teachers agreed to participate in the study. They were selected because of their interests and backgrounds in teaching mathematics and because of the other teachers' lack of preparation for teaching mathematics. Their experiences in working with emotionally disturbed children were also sufficient to allow them to modify their presently existing program to meet the needs of the investigation without causing a possible setback in the emotional behavior of the children in the study.

Both teachers were familiar with all of the students in the school since they served on evaluation teams which analyzed the students' progress. Teachers were assigned subjects after the children were given the researcher-designed diagnostic-achievement test. No attempt was made

to assign subjects to teachers in a random fashion.

Since both teachers did not have equivalent backgrounds in mathematics and in working with emotionally disturbed children, a resume of their past experiences and preparation is reported in Table 3.1 and Table 3.2. Some of the obvious characteristic differences in teacher backgrounds were: sex, teaching experience, background in working with children with special learning problems, mathematics background, and methods courses in teaching mathematics.

During the preparatory period, prior to the diagnosis of the children's mathematical skills and subsequent instruction, meetings were held with the investigator and the participating teachers. The following six points were emphasized: (1) construction of a hierarchy of mathematical skills; (2) need for and use of behavioral objectives; (3) learning and testing levels of mathematical understandings emphasizing an understanding of process and problem solving rather than mere rote or associative learning; (4) construction and administration of a diagnostic mathematics test; (5) designing and teaching instructional activities emphasizing manipulation of physical objects, active learner involvement, minimal use of paper-and-pencil activities, making mathematics fun and exciting for children; and (6) specific learning problems characteristic to emotionally disturbed children (i.e., attention span, fear of failure, negative attitudes, perceptual difficulties, mathematics and reading problems, etc.).

The investigators and the teachers were in close contact with each other throughout the study. Instruction was at the discretion of the teachers, with the investigators serving as consultants and gatherers of appropriate manipulative materials and activities. The teachers' roles were to present the diagnostic test and provide subsequent instruction based upon the performance of individual children selected for investigation. Along with the investigators and the school principal, the teachers helped select six children who met the needs of the study (i.e., mathematics inabilities spanned the hierarchy of behavioral tasks identified in Appendix A).

The teachers were asked to keep a daily log of the activities and subsequent responses of the children throughout the instructional period. All written materials and descriptive responses were gathered and kept by the teachers for the investigators to analyze via case-study at the conclusion of the study.

Students in Edgefield Lodge

There were thirty children enrolled at Edgefield Lodge at the onset of the study. All of the children had been removed from their homes and communities temporarily or permanently. Many of the children had weekend

TABLE 3.1

TEACHER EXPERIENCE AND PREPARATION

	Personal Background Information						High School Mathematics Background						College Mathematics Courses						College Mathematics Methods Courses		
	Sex of teacher	Age of teacher	Yrs. Teach. Elem. Sch. Equiv.	Yrs. Teach. Sec. Dis.	Yrs. Teach. other ch. with learn. prob.	College Degrees Held	Algebra	Geometry	Trigonometry	General Math	Others	Found. of Elem. Math	General Math	College Algebra	Trigonometry	Calculus	Other	Math. Methods	Remedial Math. Methods	Other (tests and meas.)
Teacher A	F	53	20	5	10	MS	0	1	0	0	0	3q	3q	0	0	0	0	0	0	6q
Teacher B	M	48	19	3	0	BA	1	1	0	0	0	3q	0	3q	0	0	0	6q	3q	3q

* "q" stands for quarter hours

TABLE 3.2

TEACHER EXPERIENCE AND ORIENTATIONS

	High School Psychology Oriented Courses				College Psychology Oriented Courses								Special Education Oriented Courses					
	General Psychology	Sociology	Anthropology	Others	General Psychology (undergraduate)	General Psychology (graduate)	Sociology (undergraduate)	Sociology (graduate)	Anthropology (undergraduate)	Anthropology (graduate)	Mental Hygiene	Contemporary Probs. in Chang. Society	Learning Problems (general)	Mental Retardation	Emotionally Disturb. (behavior problems)	Prevention and Correction Maladjust.	Group Dynamics	Family Dynamics
Teacher A	1s	4s	0	0	9q	30q	9q	9q	3q	0	3q	0	44q	0	24q	6q	3q 3ng	3q
Teacher B	0	0	0	0	9q	0	9q	0	0	0	3q	3q 12s	3q	0	9q	0	0	0

* "q" stands for quarter hours
"s" stands for semester hours
"ng" stands for no college credit provided

visiting privileges to their homes or foster homes; however, such an arrangement was usually not available to the newly enrolled student. The privilege of living at home was often reserved for children just prior to their returning home on a permanent living basis. The mean average of the length of a child's stay at Edgefield Lodge was 11 months. The major goal of the school was to get the children back into their homes and communities. In order to do that, it was necessary to get the students to behave in a manner that would allow them to learn and grow as emotionally stable children might, given similar living conditions.

A child's acceptance into Edgefield Lodge was contingent upon parental or guardian cooperation. This means that the whole family had to be therapeutically involved in changing the child's emotional disturbances. However, some students were wards of the juvenile court and through county welfare were assigned to foster families. Cooperation by members of the foster families was a stipulation for placement.

All students in the school displayed behaviors that disallowed them functioning in society because of abnormal emotional reactions when presented with problem solving situations. Reactions of emotionally disturbed children, when confronted with frustrating tasks, typically involved a striking out, maladaptive series of responses. Since many emotionally disturbed children are academically deficient, maladaptive behaviors are often seen as reactions to an academic setting.

While emotionally disturbed children are often identified as borderline mentally retarded, one of the stipulations for admittance to Edgefield was that they were not mentally retarded. Emotional problems existed ranging from borderline autism to children who were extremely aggressive, with a life history of petty thievery, sexual deviancy, uncontrollable tempers, and many other forms of social maladaptive behaviors. Some children had been passed back and forth from foster home to foster home since birth. Some children came from homes where they were reportedly treated as someone less than normal and were allowed to behave in deviant ways to the extent that they became dangerous to themselves and others. Some children displayed withdrawal tendencies, striking out at reportedly unpredictable moments. Many children had experienced traumatic living conditions and fought to survive. Whatever the reasons for their admittance, each child in the school was severely emotionally disturbed when he entered the school and was in some stage of behavioral change during the time the study was conducted.

Selection of Subjects

Twelve students were identified as possible subjects for the proposed arithmetic program. Criteria for selection were that the students were capable of learning skills leading up to and commensurate with an understanding of addition and subtraction of whole numbers and who had not already surpassed the terminal behaviors identified in Appendix B. The

initial screening and selection of the twelve possible subjects was conducted by the principal of the school, teachers involved in the study, and the investigators.

The twelve possible subjects were given the revised version of a researcher-designed diagnostic-achievement test. Of the twelve, six were selected by the investigators to be subjects for case investigation. The criteria for the final selection was their placement on the diagnostic-achievement test. Two of the subjects were performing at a level of mastery near the bottom of the hierarchy of arithmetic tasks, two of the subjects were performing near the middle of the hierarchy and the remaining two subjects were approximately three-fourths of the way up the hierarchy of tasks. By grouping in twos, the teachers were able to provide twenty to thirty minutes of daily instruction, rather than taking forty to sixty minutes out of their already busy schedules. It should be noted that the pairs of subjects did not progress at the same rates along the hierarchy of tasks, but were able to work cooperatively on many tasks.

Approximately half-way through the instructional phase of the study it became necessary to drop the Case-Study of Robert from the investigation because of his continued illness. The investigators, after consulting with the staff at Edgefield Lodge, decided to report his progress to the point of his excessive absenteeism and then drop his case from further investigation.

Timetable

Initial contact with Edgefield Lodge was made on December 17, 1969 when a meeting was held with Mrs. Floy Pepper, principal of the school at Edgefield Lodge. From the meeting it was decided that some of the staff at the school would be able to participate in a mathematics research project, along with subjects to be selected. Two experienced teachers were designated as the teachers who would make available approximately twenty to thirty minutes daily for special instruction in Mathematics for a manageable number of children, selected later to be six in number.

On February 3, 1970 and again on February 6, 1970, the investigators met with the two teachers and observed the children in the school setting. During those two days three students known to be beyond the range of the hierarchy of arithmetic tasks were individually tested by the investigators using the initial researcher-designed diagnostic-achievement test. From the giving of the initial test it was decided that the test was too lengthy and that the children would not remain at-task long enough to test that which was needed for diagnosis of specific arithmetic skills having been already acquired. It was felt that modifications needed to be made in the construction of the test. Some of the modifications involved breaking words down into less complex word phrases, separating

the given information from the directions teachers were to respond with and eliminating some repetitious items, with instructions to probe for understanding if there were questions in the tester's mind regarding a child's mastery of a given behavioral task.

February 6, 1970, February 19, 1970, and March 15, 1970, the investigator and the teachers involved in the study met in preparation meetings. Each session lasted longer than two hours in duration and procedures to be followed during the diagnostic and instructional phases of the program were discussed at length. On two occasions the preparation meetings lasted into the late evening. Also, the discussion was continued informally with individual teachers as further help was needed.

Between March 15, 1970 and April 3, 1970, twelve possible subjects at the school were tested using the revised version of the researcher-designed diagnostic-achievement test. This testing was done to determine the subjects to be studied and to identify the place on the hierarchy that instruction should begin for each subject selected. The teachers observed the researcher giving two of the tests, which took approximately forty-five minutes for each test, using the instructions identified in Appendix C. The researcher then observed the teachers giving one test on their own to insure consistency in the diagnosis of specific problems. During the ensuing two weeks an additional eight tests were given by the teachers to possible candidates as subjects in the study, making a total of twelve subjects having been given the revised version of the researcher-designed diagnostic-achievement test.

On April 3, 1970, instruction began in the program for the six subjects. Once a week and sometimes twice a week the school was visited and/or the teachers were contacted by the researcher to maintain consistency of instruction, to provide teaching suggestions for specific problems teachers were having with instruction, and to gather case information about the children involved in the study. Nine personal visits and four follow-up telephone calls were made to Edgefield Lodge by the investigator between April 3, 1970 and June 26, 1970.

Twelve weeks elapsed between the beginning of the instructional phase and the presentation of the post-achievement test. Between April 3, 1970 and June 26, 1970 case data regarding the instructional phase was gathered, during which time approximately twenty-two days were interrupted for various reasons explained in Table 3.3.

Table 3.3 Number of Days of Special Instruction*

Weeks	Names of Subjects: Bart	David	Karen	Cinthia	Michael	Robert
April 3	1	1	0	0	1	1
April 6-10	5	4	5	5	4	3
April 13-17	0	0	0	0	0	0
April 20-24	5	5	5	5	5	3
April 27-May 1	4	4	5	5	5	1
May 4-8	4	4	5	5	4	0
May 11-15	3	3	4	3	5	0
May 18-22	4	4	2	2	2	0
May 25-29	4	3	2	2	3	-
June 1-5	4	4	2	3	3	-
June 8-12	2	2	2	1	3	-
June 15-19	3	2	2	2	2	-
June 22-26	2	2	2	2	1	-
Total	41	38	36	35	38	8

*Periods of special instruction lasted twenty-to-thirty minutes per day. An average of thirty-eight instructional days were provided each child, spanning a twelve week period. Some of the reasons given why instruction was not provided on consecutive days were: (1) spring vacation; (2) children, teachers, or both were absent from school; (3) children were too upset to concentrate on subject matter; (4) children were excused because of teacher workshops; (5) field trips out of the school were conducted; (6) schedules of teachers were overly crowded on given days; and (7) national holidays.

Instruments Used in the Investigation

Researcher-Designed Diagnostic-Achievement Tests

The hierarchies of mathematical skills, with accompanying behavioral objectives defining each skill or understanding mentioned on the hierarchies, served as a model for the investigators while preparing the diagnostic-achievement tests. At least three exercises were included on the revised version of the test for each skill mentioned on the hierarchies. The exercises required the children to: (1) manipulate physical objects, demonstrating how they arrived at an answer for a number sentence; (2) write a number sentence numerically describing the action that the investigator or teachers have performed before them with physical objects; and (3) use of paper-and-pencil for computing the answer of the action described in a number sentence. Also, exercises to measure a child's knowledge of basic facts of the operation of addition and the inverse process of subtraction was written into the tests.

In order to determine whether or not a child had attained mastery of each mathematical task in the test, it was necessary for the teacher to have firmly in mind the criteria of the behavior under consideration. In developing the behavioral tasks identified in Appendix A and specified in Appendix B, the terminal goals were constantly under consideration. The investigator consulted numerous references, talked with knowledgeable people in the area of mathematical logic, and drew from personal experiences in order to determine major ideas which went into the mastery of the terminal objectives. The major prerequisite tasks were listed on paper and by continued modification, reorganization, and attempts to supply proof of counter examples, the sequence of tasks used for this study was developed.

Once the sequence of tasks was identified, accompanying behavioral objectives using Mager's (1962) criteria were written. Each behavioral objective contains the following component parts: (1) the identification of the desired behavior by name and/or description and specification of acceptable evidence regarding whether the learner has achieved the objective; (2) further definition of the desired behavior by describing the important conditions under which the behavior is expected to occur; and (3) the specification of criteria of acceptable performance.

Once the behavioral objectives had been written an accompanying diagnostic test was written to determine the level of mastery the learner had achieved. As mentioned previously, emphasis was placed upon oral, written and manipulative proof of comprehension.

For each behavioral task in the diagnostic-achievement test explicit directions were provided for the tester. The three components of each question were: (1) the physical objects or conditions necessary for the performance of the exercise were specified; (2) the method for manipul-

ating the physical objects was specified; and (3) the directions to be stated prior to the child performing a task were specified. A copy of the instruments may be found in Appendix C.

Attitude Scale

At the beginning of the investigation available anecdotal records were analyzed to identify the children's attitudes toward their ability to work with arithmetic tasks. Descriptive information regarding each child's responses toward arithmetic was provided in cumulative files of academic progress.

Using Osgood's (1953) semantic differential as a model, a test was prepared as an indicator of how children viewed their ability to do arithmetic at the conclusion of the study. This scale was constructed to measure the impact of the diagnostic and remedial program upon the arithmetic achievement of the five remaining emotionally disturbed children, at the end of the instructional phase of the investigation.

Twelve bi-polar items were chosen for the attitude scale. Seven blanks between each set of bi-polar items were allowed in order for the children to express strengths of feeling about the arithmetic program in which they had been engaged. The position of the positive and negative choices on the attitude scale were alternated to stop a child from expressing a completely positive or negative attitude without analyzing their feelings carefully, after having read the choices. Because they were not appropriate to arithmetic, bi-polar items six and nine were included to break up a pattern of thought by acting as distracters. Copies of the attitude scale used in this investigation are found in Appendix C.

Instructional Procedures

Nature of the Treatment

The present investigation was designed to measure the effects of a diagnostic-manipulative-remedial program upon the subjects' arithmetic performance. Also, the study was designed to measure the effect of the program on the attitudes of the children toward arithmetic. The three areas of arithmetic achievement studied were: (1) computation skills; (2) concepts; and (3) applications of mathematical processes.

The two mathematics teachers at Edgefield Lodge decided upon a workable arrangement whereby they could provide instruction for two subjects at a time. The principal of the school programmed released time into the teachers' schedules, enabling them to have regular instructional periods with the children on a daily basis, while teacher aides took over the

major responsibilities of the classroom. Children were provided with a special arithmetic period of twenty to thirty minutes per day for ten weeks. All arithmetic assignments given the children during the period of April 3, 1970 to June 26, 1970 are reported in case-study form in Chapter IV.

Table 3.3 shows the number of days each child was involved in the study. Instruction began on April 3, 1970 and ended, for all subjects receiving special arithmetic instruction, on June 26, 1970. Taking into consideration disruptions, there were approximately eight full weeks of instruction during the investigation for five of the children case-studied.

The remedial instructional program began with the lowest level skill on the hierarchy that a child could not successfully perform. When the child could exhibit the behavior specified in the behavioral objective for the skill on the hierarchy, he was assigned successively higher levels of tasks on which to work. Therefore, the child's rate of progress through the hierarchy of tasks was determined by the degree of success he achieved in irregular intervals of time on particular tasks.

Teachers designed instructional activities in cooperation with suggestions from the investigators. Instructional suggestions included the idea that textbook and workbook written activities were to be held to a minimum during the instructional phase of the investigation. Moreover, the instructional activities were to make use of physical models, manipulative aids, games, patterns of the number system, and paper-and-pencil exercises which were related to the tasks at hand. Teachers used physical models and aids as they illustrated mathematical concepts. However, even more important, was that the children manipulated physical objects and aids, the manipulation of which allowed the subjects to see the mathematical action involved in solving arithmetic problems.

At the initiation of the investigation the revised version of the researcher-designed diagnostic-achievement test was administered by the investigator and participating teachers. Twelve children were given the revised test. Six subjects were selected as a result of their placement on the tests and the need of the investigator to provide instruction for children whose mathematical proficiencies spanned the hierarchy. Furthermore, at the conclusion of the study the same researcher-designed test and an attitudinal scale were administered by the investigator to the five remaining subjects in the study. As mentioned previously, one of the subjects initially selected to be case-studied was dropped from the investigation approximately half-way through the instructional phase because of excessive absenteeism. All of the tests were hand scored by the person administering the tests and were reported by the investigator.

Teachers were required to keep an instructional file for each child involved in the case-investigation. They were instructed to keep a daily log of all mathematical activities the children were involved with in the school, in and out of the twenty-to-thirty minutes of special instruction

each day. Also, the researcher gathered background information about the subjects from cumulative progress reports on file during his visits to the school and between observations of the children. A copy of the case-study form used to guide the gathering of related information can be found in Appendix D. Along with the background information on each subject, the results of the pre- and post-researcher-designed diagnostic-achievement tests, the post-attitudinal scale, and listings of instructional activities used throughout the student are included in individually reported case-studies in Chapter IV. Instructional activity forms were used to report the activities engaged in during the program, a copy of which can also be found in Appendix D. These forms enabled the investigators to document the administration of the laboratory-oriented treatment. However, at times the teachers documented the work on scratch pieces of paper, rather than the formalized data sheets. Case data were collected during and at the conclusion of the study and were compiled by the investigators.

Case-Study Method

The case-study method of research was employed in the present investigation. A review of the literature regarding the use of case-studies failed to reveal a model which could be used in reporting information related to children's mastery or non-mastery of skills in the proposed arithmetic program. Therefore, the following organizational format was developed to organize case information.

Background Information

1. Personal Information
2. Educational History

Diagnostic-Manipulative Arithmetic Program

1. Preassessment of Arithmetic Skills
2. Instructional Program

Summary of Procedures

The study was developed through the following series of steps:

1. A practical need for the investigation of a diagnostic approach to teaching arithmetic skills to children with extreme learning problems was identified. Emotionally disturbed children were selected as subjects for the study.

2. Literature relating to mathematical disability in emotionally disturbed children, underachievement in mathematics, effects of emotional disturbances on learning, academic failure, diagnosis and remediation, use of manipulative materials, and attitude change was surveyed. Since there was no experimentally tested existing arithmetic program available for emotionally disturbed children, the investigators developed a rationale and a program for six children at Edgefield Lodge.

3. Because of the existing number of uncontrollable variables, a case-study format was adopted for reporting the result of the investigation.

4. Researcher-designed learning hierarchies for the operation of addition and the inverse process of subtraction of whole numbers were developed to aid in the diagnostic and instructional phases of the mathematics program.

5. Behavioral objectives with specific learner performance criteria were written. Each level on the hierarchies of skills was specified using observable behavior criteria.

6. Researcher-designed diagnostic tests were written and administered. The initial test was modified after trying it out on three emotionally disturbed children. Continued diagnosis using the format written into the revised researcher-designed diagnostic-achievement test was utilized throughout the instructional phase of the study. Also, the diagnostic-achievement test was used as an indicator regarding where to initiate instruction for the subjects case-studied and was again used at the conclusion of instruction as an achievement indicator.

7. Two of the five teachers at Edgefield Lodge were identified as teachers for the study.

8. Teachers were provided with three training sessions spaced over a six week period of time.

9. Six children at Edgefield Lodge were selected as subjects in the study. The revised version of the researcher-designed diagnostic-achievement test was used as a screening device.

10. Approximately eight full weeks of instruction was provided for the subjects in the study.

11. Intensive collection of data was completed and individual case-studies were compiled.

12. A descriptive analysis of the behavior and achievement patterns was compiled on the subjects by the researcher.

13. Case data were analyzed and written up in the form of summary, conclusions, recommendations, and implications for future research.

CASE DESCRIPTION AND RESULTS

Organization of the Case-Studies

To enhance readability, an organizational pattern was followed in reporting the case-studies. A brief biographical sketch is reported at the beginning of each case-study. The material relating to the instructional program under consideration is reported in detail.

For clarity, the meanings of Background Information and Diagnostic-Manipulative Arithmetic Program are described below.

Background Information

Personal Background. Each child studied was assigned an hypothetical name so that he or she cannot be readily identified. However, personal data such as sex, age, construction of family unit in which the child is a member, reported intellectual capabilities, physical stature, personal health patterns and emotional problems remained unchanged as reported to the investigators by the professional staff at Edgefield Lodge and extracted from cumulative folders. The data included a synopsis of family history, economic status, environmental influences, and other related information.

Educational History. When available, the following items of information were described regarding each child's school background: preschool experiences; entrance into school; grade in school; reaction toward school, teachers, peers, etc; and success in academic subject areas. Other related items were also described to help clarify each child's reactions toward school, especially as they related to arithmetic achievement and attitude.

Diagnostic-Manipulative Arithmetic Program

Preassessment of Arithmetic Skills. Using the revised researcher-designed diagnostic-achievement test in Appendix C and the hierarchical arrangement of behavioral tasks in Appendices A and B, mastery of specific arithmetic skills prior to the beginning of the study was identified. The results are reported in Table 4.1 for each child studied.

Instructional Program. An individualized arithmetic program was program was provided for each child and the instructional materials and activities used are described in the body of this chapter. The data were collected and reported using the following categories: (1) tasks, (2) materials and activities, (3) homeroom assignments, and (4) results.

Table 4.1: Diagnosis of Specific Arithmetic Skills Possessed by the Subjects During Pre- and Post- Instructional Periods

Skill or Task		Mastery of Skill or Task*												Test Item
		Bart Foster		David Brown		Karen Alder		Cinthia Britton		Michael Burke		Robert Myers		
	**	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	
Knowledge of Concrete Manipulative Materials and Activities		P	P	P	P	P	P	P	P	P	P	P	-	1
Discriminate and Classify Characteristics Logical to All Objectics in a Set		P	P	P	P	P	P	P	P	P	P	P	-	2-4
Discriminate and Classify Differences Not in a Set of Objects		P	P	P	P	P	P	P	P	F	P	F	-	5
Recognition of Sets		P	P	P	P	P	P	P	P	F	P	F	-	6
One-To-One Correspondence		P	P	P	P	P	P	P	P	F	P	F	-	7-8
Equivalence		P	P	P	P	P	P	P	P	F	P	F	-	9-10
Greater Than or Less Than		P	P	P	P	P	P	P	P	F	P	F	-	11

*P stands for Passing of the task and F stands for failing at the task according to behavioral criteria mentioned in Appendix B.

**Pre- refers to Pre-test of Arithmetic Competencies prior to instruction and Post- refers to Post-test of Arithmetic Competencies after instructional period.

Table 4.1: (cont'd)

Skill or Task	Mastery of Skill or Task												Test Item
	Bart Foster		David Brown		Karen Alder		Cinthia Britton		Michael Burke		Robert Myers		
	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	
Meaningful Counting	P	P	P	P	P	P	P	P	P	P	F	-	12-13
Empty Set	P	P	P	P	P	P	P	P	F	P	F	-	14
Write Meaningful Symbols for 0-9	P	P	P	P	F	F	P	P	P	F	F	-	15
Cardinality	P	P	F	P	F	F	P	P	F	P	F	-	16-18
Set Union	P	P	P	P	P	P	P	P	F	P	F	-	19-20
Set Separation	P	P	P	P	P	P	P	P	F	P	F	-	21
Commutative Property of Union	P	P	P	P	P	P	P	P	F	P	F	-	22-23
Associative Property of Union	P	P	?	P	F	P	P	P	F	P	F	-	24
Principle of Addition	P	P	P	P	P	P	F	P	F	P	F	-	25-26
Identify Element of Addition	P	P	P	P	P	P	P	P	F	P	F	-	27-28
Addition Facts with Sums less than Ten	P	P	P	P	F	P	F	P	F	P	F	-	29-30

Table 4.1: (cont'd)

Skill or Task	Mastery of Skill or Task												Test Item
	Bart Foster		David Brown		Karen Alder		Cinthia Britton		Michael Burke		Robert Myers		
	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	
Regrouping Ones to Tens	P	P	P	P	F	P	F	P	F	P	F	-	31-33
Addition Facts With Sums Greater Than Nine But Less Than Eighteen	F	P	F	P	F	F	F	P	F	F	F	-	34
Commutative Property of Addition	F	P	P	P	F	P	F	P	F	P	F	-	35-36
Associative Property of Addition	F	P	P	P	F	P	F	P	F	F	F	-	37-38
Place Value and Regrouping	F	P	F	P	F	P	F	P	F	F	F	-	39-41
Compute by Regrouping and Non-regrouping	F	P	F	P	F	F	F	P	F	F	F	-	42
Setting Up and Solving Word Problems Involving Addition	F	P	F	F	F	F	F	F	F	F	F	-	43
A. Ability to Read Problem	F	P	F	F	F	P	P	P	F	F	F	-	
B. Identify Question	F	P	F	P	F	P	F	P	F	F	F	-	
C. Identify Operation of Addition	F	P	F	P	F	P	F	P	F	F	F	-	

Table 4.1: (cont'd)

Skill or Task	Mastery of Skill or Task												Test Item
	Bart Foster Pre-	-Post-	David Brown Pre-	-Post-	Karen Alder Pre-	-Post-	Cinthia Britton Pre-	-Post-	Michael Burke Pre-	-Post-	Robert Myers Pre-	-Post-	
D. Identify Relevant Data	P	P	P	P	P	P	P	P	P	P	P	-	
E. Write Number Sentence	P	P	P	P	P	P	P	P	P	P	P	-	
F. Set Up Vertical Algorithm	P	P	P	P	P	P	P	P	P	P	P	-	
Principle of Subtraction	P	P	P	P	P	P	P	P	P	P	P	-	44-45
Addition and Subtraction as Inverse Processes	P	P	P	P	P	P	P	P	P	P	P	-	46-47
Subtraction Facts With Minuend Less than Ten	P	P	P	P	P	P	P	P	P	P	P	-	48-49
Commutative and Associative Properties of Addition	P	P	P	P	P	P	P	P	P	P	P	-	50
Subtraction Facts With Minuends Greater Than Nine	P	P	P	P	P	P	P	P	P	P	P	-	51
Subtracting Zero	P	P	P	P	P	P	P	P	P	P	P	-	52-53

Table 4.1: (cont'd)

Skill or Task	Mastery of Skill or Task												Test Item
	Bart Foster		David Brown		Karen Alder		Cinthia Britton		Michael Burke		Robert Myers		
	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	
Subtracting A Number From Itself	P	P	P	P	P	P	P	P	P	P	P	-	54-55
Compute By Regrouping Tens To Ones	P	P	P	P	P	P	P	P	P	P	P	-	56-58
Compute By Regrouping (Borrowing) and Non-regrouping	P	P	P	P	P	P	P	P	P	P	P	-	59
Setting Up and Solving Word Problems Involving Subtraction	P	P	P	P	P	P	P	P	P	P	P	-	60
A. Ability to Read Problem	P	P	P	P	P	P	P	P	P	P	P	-	
B. Identify Question	P	P	P	P	P	P	P	P	P	P	P	-	
C. Identify Process of Subtraction	P	P	P	P	P	P	P	P	P	P	P	-	
D. Identify Relevant Data	P	P	P	P	P	P	P	P	P	P	P	-	
E. Write Number Sentence	P	P	P	P	P	P	P	P	P	P	P	-	
F. Set Up Vertical Algorithm	P	P	P	P	P	P	P	P	P	P	P	-	
Principle of Multiplication		P											61-62

Tab;e 4.1: (cont'd)

Skill or Task	Mastery of Skill or Task						Test Item
	Bart Foster Pre-Post-	David Brown Pre-Post-	Karen Alder Pre-Post-	Cinthia Britton Pre-Post-	Michael Burke Pre-Post-	Robert Myers Pre-Post-	
Multiplication Facts	P						63-64
Multiplication By Zero	P						65-66
Multiplication By One	P						67-68
Commutative Property of Multiplication	P						69-70
Associative Property of Multiplication	P						71
Distributive Property of Multiplication	P						72-73

In order to make the data more readable and to conserve space the arithmetic instruction provided for the children is reported by the week, rather than the day. Each child's instructional program is analyzed at the conclusion of each case-study.

Case Study of Bart

Biographical Sketch

Bart Foster was an eleven-year old whose aggressive behavior and defiant attitude made it extremely difficult for him to concentrate on academic subjects. He was supposed to have been in the fifth grade but spent two years in the first grade. Because of this he was one of the largest children in his classrooms. Bart was described as a tall, well-built, coordinated child with an inclination to manipulate those with whom he interacted. Some of the adjectives used to describe Bart's behavior were: hyperactive, manipulating, restless, demanding, hostile, anxious, fearful, disobedient, angry, uncooperative, short attention, lying, stealing, delinquent, easy tempered, and nervous.

At the time of Bart's intake interview for Edgefield Lodge he was living with his mother, his mother's employer, and the employer's older daughter. Bart's natural father and his mother had divorced when he was about three years old. Shortly thereafter, Bart, his younger brother Tony, and his mother moved to Portland, Oregon. Mrs. Foster had difficulty obtaining employment and when Tony developed a crippling physical defect, requiring intensive care, she decided to place both boys in foster homes. Tony was still in a foster home at the time of this study. However, Bart was only separated from his mother for a year. The experience apparently created a fear of not belonging, so in order to gain attention Bart proceeded to play a role of power seeking, manipulating his mother and other authority figures at every opportunity. Shortly before this study was initiated, Mrs. Foster and Mr. Herman Armstrong, the employer, were married. Thus, a more firm home situation was established, clarifying many of the concerns of the professional staff at Edgefield Lodge regarding the possibility of corrective therapy for Bart's emotional and academic behaviors.

Academic performance was difficult for Bart. At the time this study was initiated he was working at a high second or low third grade level in both reading and mathematics. Bart had difficulty remaining at-task. However, he entered Edgefield Lodge on August 10, 1969, and from that time to the beginning of the study had progressed from a state of complete reliance on the teacher for assistance in his subjects to a point where he felt he could attempt work in many school subjects. The one subject Bart expressed the most dislike for and fear of was mathematics. He was unsuccessful at it. His reading ability was poor and he had difficulty concentrating on what he read.*

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* NOTE: Detailed background information has been removed to preclude the possibility of the subjects being identified at a future time. Qualified individuals desiring further information may contact the investigator directly.

Diagnostic-Manipulative Arithmetic Program

Preassessment of Arithmetic Skills. Even though it was indicated that Bart was performing at a third grade level in arithmetic, the results from the revised researcher-designed diagnostic-achievement test, reported in Table 4.1, showed that he had not mastered many skills related to addition and subtraction of whole numbers by the onset of the study. Referring to the mathematics hierarchies in Appendix A, Bart lacked speed and accuracy in the more complex addition facts, lacked an understanding of the commutative and associative properties of addition, and lacked an understanding of the place value and regrouping necessary to compute addition problems. Also, he demonstrated inability to solve word problems involving addition and subtraction.

Before the study began, Bart understood little about subtraction, although he seemed to understand what the operation involved. He had only a vague understanding that addition and subtraction are inverse processes and he knew very few subtraction facts. Bart's work in addition and subtraction was seemingly at a rote, rule oriented, level of performance.

Instructional Program. Instruction began for Bart on April 3, 1970. Teacher B was his teacher for the special arithmetic sessions, providing forty-one instructional periods over twelve weeks. Bart and another child, David, received instruction together and seemed to work harmoniously.

Figure 4.1 is a descriptive account of the arithmetic program provided for Bart during the instructional phase of the study. Also, Bart's attitude toward his ability to do arithmetic is tested by his responses on the semantic differential attitude scale in Scale 4.1. He was asked to respond to the attitude scale on the final day of special arithmetic instruction, June 26, 1970. Furthermore, post-instructional arithmetic skills are reported in Table 4.1. The instructional program and Bart's attitude changes are analyzed at the conclusion of the case study.

Bart demonstrated mastery of all arithmetic skills identified in the addition and subtraction hierarchies in Appendix A. He also spent considerable time working with multiplication concepts on homeroom assignments. However, it was not until the last few weeks of the instructional period that Bart related multiplication with addition. Perhaps much of the time used in homeroom assignments emphasizing ideas and computation related to multiplication could well have been spent reinforcing addition and subtraction concepts, especially during the first half of the instructional period.

Post-assessment of arithmetic skills indicated that Bart had attained a model for multiplication and that he had a mastery of the identity element of multiplication. However, he performed poorly on multiplication facts and related properties such as the zero property, commutative property, associative property, distributive property, and other higher level tasks.

Bart's attitude toward arithmetic improved considerably during the study. Whereas he expressed only negative comments about anything to do with arithmetic during the pre-testing, he demonstrated a slightly positive reaction to his ability to do arithmetic during the post-testing phase. Bart's homeroom teacher mentioned that he spoke highly of his special instructional periods. Many of the concrete manipulative ideas learned during the special sessions were later used to check his homeroom assignments. By using concrete illustrations Bart was able to provide visual proof for the arithmetic actions with which he was involved.

Scale 4.1: Bart's Attitude Toward His Ability To Do Arithmetic Following Instruction

Big	X							Little
Unhelpful							X	Helpful
Old	X							Young
Strong	X							Weak
Powerless							X	Powerful
Deep				X				Shallow
Nice	X							Awful
Slow	X							Fast
Sour				X				Sweet
Alive				X				Dead
Good	X							Bad
Quiet	X							Noisy

Figure 4.1 Instructional Program Provided for Bart
From April 3, 1970 to June 26, 1970

Week	No. of Days	Tasks	Materials and Activities	Homework Assignment	Results
April 3	1	addition facts with sums greater than nine and less than eighteen	made flash cards to build accuracy and speed in addition facts	none	Bart needed more practice on addition facts
April 6-10	5	same as April 3 associative and commutative properties of addition	made flash cards to build accuracy and speed in addition facts 100 addition facts on a timed test mental arithmetic to build speed and accuracy in simple addition and subtraction facts; winner got to be caller block cubes and paper-and-pencil; first teacher manipulated and Bart wrote number sentences, then teacher wrote number sentences and Bart manipulated blocks to demonstrate a + b = b + a and (a + b) + c = a + (b + c)	drill and practice on vertical addition and subtraction algorithms	Bart needed more work on addition facts, especially those involving mental regrouping. Drill and Practice on vertical addition and subtraction algorithms seemingly had produced a rote understanding of processes. When questioned about why he did what he did he was unable to explain. Also, very little detail using borrowing and carrying was followed. Bart counted on his fingers to arrive at the combination needed. Bart needed more work on the commutative and associative properties of addition.
April 13-17	0	none (Spring vacation)	none	none	none
April 20-24	5	same as April 6-10 addition and subtraction as inverse processes subtraction facts with minuends less than or equal to nine	addition flash cards to build speed and accuracy made flash cards to build accuracy and speed in subtraction facts writing number sentences related to manipulation of block cubes, etc. mental arithmetic using operations of addition and subtraction 100 addition facts on timed tests 45 subtraction facts on timed test 10 x 10 matrix for addition with blank spaces for Bart to fill in demonstrating commutative property	story and written problems involving parts of wholes (fractions) drill and practice on vertical subtraction problems work was assigned from textbooks involving introductory arrays leading to multiplication as repeated addition	Bart mastered addition facts with some sums greater than nine and less than eighteen. Bart mastered addition and subtraction as inverse processes. Bart needed more work on subtraction facts with minuends less than or equal to nine. Bart mastered the commutative and associative properties as related to subtraction. There was confused ideas involving problems concerning parts of wholes. Drill and Practice on higher order addition and subtraction was seemingly confusing and appeared to encourage counting and finding answers by rote. Bart needed help on place value when dealing with vertical subtraction problems

Figure 4.1 (con't)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
			block cubes, sets of pencils and other readily available materials along with paper-and-pencil; Bart manipulated objects after teacher wrote and talked about relationships between addition and subtraction		Bart realized that multiplication can be represented by repeated addition.
April 27-May 1	4	subtraction facts with minuends less than or equal to eighteen subtraction where borrowing is not required	subtraction flash cards 100 subtraction facts on timed tests mental arithmetic using operations of addition and subtraction subtraction problems from a textbook not requiring regrouping to work on speed and accuracy simple story problems involving subtraction from worksheet	story and written problems involving parts of wholes	Bart needed more work on subtraction facts greater than nine and less than or equal to eighteen. Bart mastered subtraction facts with minuends less than or equal to nine. Bart mastered computing subtraction problems where borrowing was not required. Bart needed more work on computing subtraction problems requiring regrouping.
May 4-8	4	subtraction facts with minuends greater than nine and less than or equal to eighteen subtraction problems emphasizing place value place value and regrouping in addition	subtraction flash cards to build speed and accuracy 100 subtraction facts on timed tests textbook materials requiring Bart to solve vertical subtraction problems involving regrouping simple story problems involving subtractions; SRA drill and practice exercises bundles of pipe cleaners, place value pocket chart, dollar bills and stacks of dimes and pennies; Bart manipulated objects after teacher wrote addition and subtraction problems on chalkboard requiring regrouping and vice versa	exercises from textbook requiring use of simple multiplication facts less than or equal to four story and written problems involving parts of wholes	Bart mastered subtraction facts with minuends greater than nine and less than or equal to eighteen. Bart needed more help on subtraction problems emphasizing place value. Bart needed more help on addition problems requiring an understanding of place value. Bart had trouble with multiplication facts involving the zero property.

Figure 4.1 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homework Assignments	Results
May 11-15	3	compute by regrouping and non-regrouping in addition three digit subtraction problems requiring regrouping compute by regrouping and non-regrouping in subtraction	addition and subtraction flash cards addometer and seven pronged abacus; Bart manipulated objects to solve addition and subtraction problems requiring regrouping drill and practice from text materials using horizontally presented numbers to be aligned vertically and solved	same as May 4-8	Bart needed more work on computing by regrouping and non-regrouping for addition and subtraction. Bart had trouble with the multiplication property of zero and the identity element of multiplication.
May 18-22	4	story problems mixing operations of addition and subtraction place value of digits in numerals	worksheets with story problems and pencil-and-paper Bart wrote up his own story problems involving addition and subtraction of whole numbers abacus; Bart used abacus to demonstrate understanding of story problems made up by teacher mental arithmetic using operations of addition and subtraction	filling in component parts of a 5 x 5 matrix, demonstrating relationships about multiplication facts less than or equal to five using workbook and textbook exercises; multiplication and division as inverse processes using workbook and textbook exercises; division as repeated subtraction	Bart needed more work on orally presented and written story problems, differentiating whether addition or subtraction should be used. Bart needed more help on place value of digits in numerals and the necessity to align digits in columns and rows. Bart still had problems with the zero property of multiplication. The process of division of whole numbers was apparently not understood since Bart wrote down mostly nonsense answers. Bart made numerous errors with simple multiplication facts. Bart lacked an understanding of the communicative property of multiplication. Bart gave many nonsense answers in multiplication and division. Bart had become quite discriminant regarding the operations story problems asked him to use. At times he needed help with reading the words. He got confused with subtraction and division. Bart had not mastered place value
May 25-29	4	story problems mixing addition and subtraction situations place value of digits in numerals and appropriate alignment in setting up vertical addition and subtraction algorithms	worksheets with addition and subtractive situations on same page same as May 18-22	exercises in textbook requiring use of simple multiplication facts less than or equal to six story and written problems involving parts of wholes simple division exercises such as $15 \div 5 = \square$ and $4 \overline{)16}$	

Figure 4.1 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
June 1-5	4	place value compute by regrouping and non-regrouping in addition compute by regrouping and non-regrouping in addition (carrying) compute by regrouping tens to ones in subtraction compute by regrouping and non-regrouping in subtraction (borrowing)	textbook and workbook materials with periodic concrete illustrations using physical objects from the teacher	story problems in multiplication and division from textbook exercises in textbook requiring use of simple multiplication facts less than or equal to four multiplication facts demonstrated by use of arrays of objects on a numberline and on an abacus	Bart became quite proficient in his understanding of place value in addition and subtraction situations. He needed a little more reinforcement. Bart needed help on multiplication and division. He appeared frustrated and frequently wadded his paper when got his written assignments back from his home room teacher. He still had problems with the identity element of multiplication.
June 8-12	2	same as June 1-5	same as June 1-5	exercises in textbook requiring use of simple multiplication facts less than or equal to five	same as June 1-5
June 15-19	3	compute by regrouping and non-regrouping (borrowing) story problems differentiating different types of subtraction situations	paper-and-pencil exercises; from textbook and later Bart making up his own story problems and finding solutions	none	Bart demonstrated mastery of computing by regrouping (borrowing) and non-regrouping in subtraction situations. Bart became more proficient in differentiating subtractive situations.
June 22-26		review number facts in addition and subtraction take achievement test	mental arithmetic using addition and subtraction revised researcher-designed diagnostic-achievement test	none	see post-test data in Table 4.1

Case Study of David

Biographical Sketch

David Brown was ten years old when this study was conducted. He was described as an incorrigible child because of his aversive behavior at home, at school, and in the community. David's mother reported him to be increasingly difficult to reason with. His public school classroom teacher had dismissed him permanently for his contant disruptions, inability to follow directions, and irregular attendance. Twice in the months just prior to his entrance to Edgefield Lodge David had been placed in a juvenile home because he had run away from home and had been caught stealing.

David was described as a child having extreme eye problems. He had gone without glasses most of his life. David was of medium height, thin, and somewhat uncoordinated physically. Some of the adjectives used to describe David and his behavior were: restless, sneaky, little imp in cowboy boots, disobedient, lying, fearful, hyperactive, uncooperative, physically immature, emotionally immature, short attention span, run away, etc.

David was from a broken home. His mother and father were divorced when he was around five years of age. David has two older brothers and a three year old sister. Mrs. Brown had attempted to hold the boys close to home but had apparently not succeeded. David associated with children older than himself and had performed delinquent acts to prove his ability to be one of a gang of older boys. His home environment seemed to stabilize after his entrance to Edgefield Lodge. Mrs. Brown married Mr. Charles Ratcliff and they both worked cooperatively with the staff at the school to modify the reinforcing behaviors of the family members, thus encouraging David to control his emotional behaviors and to become more responsible for his actions. Previously, Mrs. Brown had not enforced what she asked David to do and as a result he began listening to her demands less and less.

When he entered Edgefield Lodge on November 23, 1969 David was a fifth grade child working at a second grade level in arithmetic. When this study was initiated David reportedly was capable of remaining at-task for extended periods of time if he was motivated. Arithmetic was the one subject with which he had met constant failure, but he had a real desire to improve his skills. Since his ability to read was poor, David had difficulty following instructions without having them read to him.

Diagnostic-Manipulative Arithmetic Program

Preassessment of Arithmetic Skills. David was performing at a second

grade level in arithmetic. The results from the revised researcher-designed diagnostic-achievement test, reported in Table 4.1, showed that he had not mastered many skills related to addition and subtraction of whole numbers by the onset of the study. Referring to the mathematics hierarchies in Appendix A, David lacked speed and accuracy with basic addition and subtraction facts, lacked the ability to write meaningful symbols from zero through nine (words written out), lacked an understanding of the inverse relationship of addition and subtraction, lacked place value and regrouping skills, lacked an understanding of the number properties as they pertain to subtraction, and lacked the ability to set up and solve word problems. Much of David's work in addition and subtraction was seemingly at a rote, rule oriented, level of performance.

Instructional Program. Instruction began for David on April 3, 1970. Teacher B was his teacher for the special arithmetic sessions, providing forty instructional periods over twelve weeks. David and another child, Bart, received instruction together and seemed to work harmoniously.

Figure 4.1 is a descriptive account of the arithmetic program provided for David during the instructional phase of the study. Also, David's attitude toward his ability to do arithmetic is tested by his responses on the semantic differential attitude scale in Scale 4.2. He was asked to respond to the attitude scale on the final day of special arithmetic instruction, June 26, 1970. Furthermore, post-instructional arithmetic skills are reported in Table 4.2. The instructional program and David's attitude changes are analyzed below.

David demonstrated mastery of all tasks identified in the hierarchy of addition and subtraction of whole numbers in Appendix A, except the following: "subtraction facts with minuends greater than nine and less than eighteen", "compute by regrouping (borrowing) and non-regrouping", and setting up and solving word problems involving addition and subtraction word problems. However, he was able to demonstrate mastery of other tasks necessary in order to solve addition word problems. He was able to identify the question and identify the necessary operation, but was unable to continue. The above mentioned skills are reported in the post-assessment of arithmetic tasks in Table 4.1.

At the conclusion of the study David responded to the attitude scale in a somewhat positive manner. However, there was some indication that he still harbored negative feelings about his ability to achieve in arithmetic. David's attitude toward arithmetic improved considerably from what it was at the beginning of the study since at that time there were few indications of positive feeling about arithmetic. His attitude toward arithmetic at that time was avoidance whereas at the conclusion of the study, when confronted with an exercise or activity within his realm of understanding, he approached it in a problem solving way. Sometimes he elicited the help of the teacher to do problems he had previously answered correctly. According to David's homeroom teacher, David had made much improvement in mathematics during the period of this study, not only in content but also in his willingness to work without the help of the teacher.

Scale 4.2: David's Attitude Toward His Ability To Do Arithmetic Following Instruction

Big	X							Little
Unhelpful			X					Helpful
Old	X							Young
Strong		X						Weak
Powerless					X			Powerful
Deep						X		Shallow
Nice					X			Awful
Slow	X							Fast
Sour							X	Sweet
Alive				X				Dead
Good	X							Bad
Quiet				X				Noisy

Figure 4.2 Instructional Program Provided for David
From April 3, 1970 to June 26, 1970

Week	No. of Days	Tasks	Materials and Activities	Homework Assignments	Results
April 3	1	addition facts with sums less than or equal to eighteen	made flash cards to build accuracy and speed in addition facts	Addition facts and simple subtraction facts on worksheets	David needed more practice with addition and subtraction facts. At times he forgot to take note of the operation signs.
April 6-10	4	same as April 3	flash cards to build accuracy and speed in addition facts	used numberline exercises in workbook and block cubes for manipulation; worked on subtraction as the undoing of addition	David needed more practice with addition and subtraction facts to build speed and accuracy
		subtraction facts with minuends less than or equal to nine	100 addition facts on timed tests		
		addition and subtraction as inverse processes	45 subtraction facts on worksheets	paper-and-pencil exercises in writing out number sentences in longhand then numerical sentences such as 2 + 4 = 6	David was able to see that addition and subtraction are inverse processes when he had the physical referent. At times he neglected to notice the operation signs.
		commutative property of addition	block cubes and paper-and-pencil first teacher manipulated and David wrote number sentences, then teacher wrote number sentences and David manipulated blocks to demonstrate a + b = b + a and a + b = ?, etc. -- also, a + b = b + a	worksheets on renaming whole numbers with problems such as 9 - 1 = 2 + 6	David mastered the commutative property of addition.
			mental arithmetic to build speed and accuracy in simple addition and subtraction facts		David needed more work on writing appropriate number sentences in longhand and in numerical symbols.
					David caught on to the math problems dealing with the number line.
April 13-17	0	none	none	none	none
April 20-24	5	addition facts with sums less than or equal to eighteen	flash cards to build accuracy and speed in addition	worked with numberline, demonstrating problems such as 9 - 2 = 7 and 2 + 4 + 3 = 9	David mastered addition facts with sums less than or equal to nine.
		subtraction facts with minuends less than or equal to eighteen	made flash cards to build accuracy and speed in addition facts	drill and practice on vertical addition and subtraction problems	David needed more work on subtraction facts. He was close to mastery of subtraction facts with minuends less than or equal to nine.
		addition and subtraction as inverse processes	mental arithmetic to build speed and accuracy in simple addition and subtraction facts	workbook assignments writing responses to problems requiring David to identify the number of bundles of ten pencils and single pencils, with accompanying number sentences such as: 1 ten 6 ones 16 in all; 1 ten 7 ones 17 in all; etc. -- also, 20 + 0 = 20, 3 tens and 0 ones = 30, etc.	David's speed and accuracy with addition and subtraction facts has improved greatly, especially with sums and minuends less than or equal to nine.
		regrouping ones to tens in addition	writing number sentences as teacher manipulated block cubes, etc., and given number sentences David manipulated objects		
			written exercises of addition requiring regrouping of ones to tens; also, demonstrated results using block cubes and abacus	10 x 10 matrix for addition facts, David filled in missing sums	At times David had difficulty concentrating on his work. However, when he tried he most often was successful.

Figure 4.2 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
April 27-May 1	4	addition facts with sums greater than nine and less than or equal to eighteen	addition and subtraction flashcards to build speed and accuracy		David mastered addition and subtraction as inverse processes.
		addition facts with minuends less than or equal to eighteen	100 addition facts on timed tests		David needed more work on writing longhand sentences; however, he was quite successful in writing appropriate number sentences.
		commutative and associative properties of addition as related to subtraction	100 subtraction facts on timed tests		David was able to correctly fill in the blanks of the 10 x 10 matrix for addition facts.
			mental arithmetic using both addition and subtraction facts		David mastered regrouping ones to tens in addition.
			written subtraction problems where borrowing was not required; David was asked to demonstrate what he did on his paper by manipulating an abacus and a set of block cubes.	worksheets having to do with beginning notions of multiplication--multiplication as repeated addition--fill in the blanks; e.g., there are 4 sets of things. There is 1 thing in each set. There are 4 things altogether. Also, 3 ( ) + 3 ( ) + 3 ( ) = 6; 2 + 2 + 2 = 6; 3 twos = 6. Also, 4 + 4 + 4 + 4 = 16; 4 fours = 16; 4 times 4 = 16. Also, arrays of squares such as 3 x 5 = 15.	David mastered addition facts with sums greater than nine and less than eighteen.
			David was asked to demonstrate his understanding of the commutative and associative properties by manipulating block cubes, etc., to represent subtraction and addition number sentences written on paper; also to write number sentences indicating mathematical actions which Teacher B performed using block cubes, pencils, straws, pipe cleaners, etc.		David needed more work on subtraction facts -- 96/100 in four minutes.
					Subtraction where borrowing is not required was mastered.
					David understood and correctly described how the commutative and associative properties of addition are not applicable to subtraction.
					David correctly responded to worksheet assignments on multiplication as repeated addition.
May 4-8	4	subtraction facts with minuends less than or equal to eighteen	subtraction flash cards to build speed and accuracy	worksheets with multiplication facts less than or equal to five	David needed more help with subtraction facts with minuends greater than nine and less than eighteen.
		place value and regrouping in addition	100 subtraction facts on timed tests	story problems involving simple multiplication facts	David needed more work on subtraction problems requiring regrouping.
		subtraction problems emphasizing place value	textbook materials requiring David to solve vertical subtraction problems involving regrouping	worksheets involving addition of three digit numbers--also, story problems with addition and subtraction situations	David needed more work on simple multiplication facts and fractional parts of wholes. At times his answers appeared to be at random.
			simple story problems involving subtraction	worksheet involving lengths and widths of rectangles in inches.	David had difficulty staying in his seat and needed social reinforcement to keep going. He was not belligerent, just active.

Figure 4.2 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
			drill and practice exercises from SRA test.		
			demonstrations of place value where David manipulated objects and wrote appropriate number sentences--bundles of pipe cleaners in ones, tens and hundreds used; place value pocket chart with numerals used; dollar bills, stacks of dimes and pennies used; abacus used.		
May 11-15	3	subtraction facts with minuends less than or equal to eighteen	subtraction flash cards to build speed and accuracy	worksheets with addition, subtraction and multiplication linearly aligned number facts requiring David to differentiate between operation sign such as 9 + 9 = 18, 11 - 7 = 4, and 3 x 4 = 12 were presented	David needed more work on subtraction facts with minuends less than or equal to eighteen, especially where the differences are quite large.
		subtraction problems emphasizing place value	100 subtraction facts on timed tests		
		compute by regrouping and non-regrouping in addition	Teacher S gave subtraction problems requiring regrouping and David manipulated objects to demonstrate the arithmetic action; abacus, place value pocket chart and addometer were used; addition problems requiring regrouping was reviewed	continued work with simple multiplication problems requiring David to associate arrays of one inch squares with multiplication sentences such as 2 x 5 -- the most complex array was an 5 x 8 array	David was able to handle addition problems requiring regrouping and non-regrouping; however, perhaps a little more drill and practice would enable him to perform the mathematical actions in a less rote fashion. He apparently does understand place value but had a difficult time remaining at-task long enough to formally organize a verbal explanation. Social reinforcers such as praise, etc. were used to bring him back to the task at hand.
				worked on time clocks for precision	David had a difficult time handling the mixed operation exercises. At first he did not know what to do with linearly aligned problems. He then got the operation signs of "+", "-", and "x" interchanged.
				worked on combinations of coins	David had little difficulty working with time clocks. However, he did have difficulty with money combinations of coins.
				simple story problems dealing with multiplication and identifying unknown quantities: e.g., x x 4 = 16 and 4 = ?	David's reading ability had improved greatly since his arrival at Edgefield Lodge. However, he still had problems reading story problems. When the problems were read to him, in many cases he could solve the problems without asking any further questions.

Figure 4.2 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
May 12-22	4	Subtraction facts with minuends less than or equal to eighteen compute by regrouping and non-regrouping in addition subtracting zero from another number subtracting a number from itself compute by regrouping tens to ones in subtraction set up vertical addition and subtraction algorithms	flash cards to build speed and accuracy in addition and subtraction 100 addition facts on timed tests 100 subtraction facts on timed tests worksheets requiring David to add problems requiring regrouping and non-regrouping--also, several problems were given to David with which he had to align the numerals himself and then solve using block cubes and the abacus, David was asked to perform problems that required him to subtract zero and a number from itself, then to write a number sentence describing what he had done worksheets were given David which required him to regroup tens to ones using the process of subtraction--block cubes were used to demonstrate the mathematical action	worked on problems asking him to identify sets of consecutive even and odd numbers worked on problems asking him to identify unknown quantities in a number sentence involving addition and subtraction number facts he worked on addition and subtraction story problems	David needs more work on subtraction facts with minuend less than or equal to eighteen. There were days when David had difficulty concentrating on the number facts in addition and simple facts in subtraction, earlier identified as having been mastered. David had difficulty reversing numerals in subtrahend and minuend and then subtracting; again it would suggest he was not paying attention or that he does not fully understand the regrouping process and/or does not have a mature understanding of place value. Subtracting zero and subtracting a number from itself were mastered. David was able to correctly allign both subtraction and addition components of word problems in vertical algorithms. David appeared to understand the differentiation of even and odd numbers.
May 25-29	3	subtraction facts with minuends less than or equal to eighteen review addition facts less than or equal to eighteen compute addition and subtraction problems requiring regrouping and non-regrouping	flash cards for accuracy and speed 100 subtraction facts on a timed test 100 addition facts on a timed test mental arithmetic, mixing addition and subtraction number facts worksheets requiring David to regroup quantities in both addition and subtraction; the abacus, pipe cleaners were used for David to describe what he had done on a few problems	worksheets requiring regrouping with addition and subtraction in vertical algorithm form were given to David to solve worksheets with story problems involving addition, subtraction, and multiplication were given to David to solve worksheets requiring David to identify place value of thousands, hundreds, tens, and ones were given to solve worksheets asking David to fill in unknown quantities using addition, subtraction, and multiplication	Subtraction facts with minuends less than or equal to eighteen needed more work. David appeared to have a block when it came to subtraction problems involving differences of 7, 8, and 9. Mastery was again demonstrated for addition facts. David appeared to have an understanding of place value. However, he still solved problems in addition and subtraction in a rote fashion. Story problems were still difficult because of readability. David had problems differentiating subtraction problems readily and made wild hypotheses regarding how to solve them.

Figure 4.2 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
		story problems mixing addition and subtraction situations	worksheet with story problems having addition and subtraction situations were presented	worksheets involving parts of wholes were given to David to solve	David had difficulty working problems involving parts of wholes such as ½ of 12 is ⅓.
June 1-5	4	subtraction facts with minuend less than or equal to eighteen	flashcards for accuracy and speed	worksheets demonstrating arrays for multiplication as repeated addition	David forgot many of the subtraction facts.
		compute addition and subtraction requiring regrouping and non-regrouping	100 subtraction facts on a timed test	textbook assignment involving sums and differences, with unknown quantities	David finally had given indications of mastery regarding regrouping in addition and subtraction of whole numbers. He was able to explain what takes place regarding place value with reference to vertical addition and subtraction algorithms.
		story problems mixing addition and subtraction	mental arithmetic, mixing addition and subtraction	textbook assignments involving story problems with addition and subtraction	Story problems involving addition were easier; however, subtraction was still a dilema.
			worksheets requiring David to regroup quantities in both addition and subtraction--the place value pocket chart and the abacus were used to demonstrate manipulation of digits		David needs more help on multiplication as repeated addition.
			worksheets with addition and subtraction story problems were presented		Again, David had difficulty remaining at task.
June 8-12	2	subtraction facts with minuend less than or equal to eighteen	flash cards for subtraction facts	none	There was substantially no improvement with subtraction facts. He apparently has a desire to know them but does not see computational shortcuts.
		story problems mixing addition and subtraction	100 subtraction facts test		David can readily identify addition problems but had difficulty realising what to do with subtraction problems. He apparently can identify whether or not a problem involves subtraction but gets hung up when non-essential items are interjected into a problem.
			basically paper-and-pencil exercises, taken from textbook and workbook material--when concrete illustrations were necessary Teacher B either used illustrations or physical materials		David had difficulty remaining at task and mentioned that his friends at home are out of school.
			mental arithmetic, mixing addition and subtraction		
June 15-19	2	subtraction facts with minuend less than or equal to eighteen	same as June 8-12	same as June 8-12	
		story problems mixing addition and subtraction			

Figure 4.2 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Classroom Assignment	Results
June 22-26	4	subtraction facts with minuend less than or equal to eighteen story problems mixing addition and subtraction review number facts in addition and subtraction take achievement test	100 subtraction facts test David became involved in writing and setting up his own story problems, beginning to grasp the idea that subtraction problems can be stated in more than one way. mental arithmetic revised researcher-designed diagnostic-achievement Test	David was allowed to continue his interests in setting up his own story problems	David needs more help with more difficult subtraction facts. David appeared to be turned on with writing his own arithmetic problems. He needs more help. See post-test data in Table 4.1.

Case Study of Karen

Biographical Sketch

Karen Alder was a ten year old, fifth-grade child from an upper middle-class home setting. As an infant, because of her slow motor and speech development, Karen had been diagnosed as having cerebral palsy and as being intellectually retarded. Her condition was shown later to be slow maturation. However, this resulted in Karen being treated as someone who was grossly incompetent. In order to gain acceptance she adopted infantile behavior such as crying, biting, screaming, and kicking. She displayed these behaviors at home and later at school. She had been refused entrance into kindergarten at the age of five because of her immature physical, emotional, and social behavior; she later matured physically to where she was taller than all, and perhaps as coordinated as most, of the age equivalent boys at Edgefield Lodge. Other adjectives used to describe Karen were: emotionally immature, social conscious, tomboyish, extremely aggressive, fear of failure, harm-avoidance, competence-anxiety, somewhat optimistic about capabilities, braggert, eye-to-eye contact avoidance, dependent upon adults, attention seeking, hyper-feminine, honest, slow worker, etc.

Karen's interactions with people continually reinforced her infantile behavior. Her two older sisters avoided confrontations with her by not treating her as an equal. Whenever Karen came around them they reportedly talked to her as if she were a much younger child; excluding her from girl-talk conversations, giving her demands rather than requests, etc. Her younger brother apparently enjoyed intimidating Karen to the point of fighting with her constantly. Her father teased her in fun ways, nevertheless, causing Karen to get upset over trivial matters. Mrs. Alder was overprotective of Karen and defended her against everyone, as did the whole family when confronted outside the family setting. Karen had continued her unpredictable anti-social behaviors in school and was constantly being placed in frustrating circumstnaces; having extreme difficulty maintaining friends, constantly saying and doing socially unacceptable things to her peers and teachers, constantly fighting to save face, and constantly getting further behind in her academic subject areas because of her inability to concentrate. After Karen's entrance to Edgefield Lodge on July 8, 1969 much of the therapy treatment time was used trying to rid her of aggressive acts.

When this study was initiated Karen had accepted a feminine role and had adopted many socially accepted behavior patterns; i.e., helping in the kitchen, keeping her room neat and tidy, wearing dresses, taking an interest in music and other fine arts, etc. She gradually was able to gain attention from significant people in her life by doing other things besides rough-housing. Cinthia was reported to be Karen's first girl friend. Both girls gained experientially from the relationship because they were able to share girl-talk with each other and challenge each other to respond in

socially acceptable modes of behavior.

Karen's academic performance improved considerably from the time she entered the school to the time that this study was initiated. Just prior to her acceptance to Edgefield Lodge Karen's mother and public school teacher had come to the conclusion that Karen had more potential than previously thought. For instance, she was able to memorize long lists of popular songs, the words to those songs, and performing artists - a clear indication that Karen could remember things of importance to her. Also, Karen had the ability to recall names and details of events and to draw pictures with exceptional detail. However, mathematics continued to be Karen's most difficult challenge, according to her homeroom teacher at Edgefield Lodge. Reportedly, she was working at a high first or low second grade level in arithmetic, partially because of her poor reading habits, but more because she did not understand the mathematical actions involved.

Diagnostic-Manipulative Arithmetic Program

Preassessment of Arithmetic Skills. The results from the revised researcher-designed diagnostic-achievement test, reported in Table 4.1, showed that Karen had not mastered many skills related to addition and subtraction of whole numbers by the onset of the study. Referring to the mathematical hierarchies in Appendix A, Karen was not able to write out the names of the number properties of zero to nine (at times she did not even recognize the words in print). She had not mastered tests for "cardinality", "set union", "set separation", "principle of addition", "principle of subtraction", and higher-level tasks, including ability to set up word problems and compute. Karen was able to read the story problems, even though often she did not know what she had read.

Instructional Program. Instruction began for Karen on April 6, 1970. Teacher A was her teacher for the special arithmetic sessions, providing thirty-six instructional periods over twelve weeks. Karen and another child, Cinthia, received instruction together and seemed to work cooperatively.

Figure 4.1 is a descriptive account of the arithmetic program provided for Karen during the instructional phase of the study. Also, Karen's attitude toward her ability to do arithmetic is tested by her responses on the semantic differential attitude scale in Scale 4.3. She was asked to respond to the attitude scale on the final day of special arithmetic instruction, June 26, 1970. Furthermore, post-instructional arithmetic skills are reported in Table 4.3. The instructional program and Karen's attitude changes are analyzed at the conclusion of the case study.

Scale 4.3: Karen's Attitude Toward Her Ability To Do Arithmetic Following Instruction

Big	___	X	___	___	___	___	___	Little
Unhelpful	___	___	___	___	X	___	___	Helpful
Old	___	___	___	___	X	___	___	Young
Strong	___	___	___	___	___	X	___	Weak
Powerless	___	___	X	___	___	___	___	Powerful
Deep	___	___	___	___	___	X	___	Shallow
Nice	___	___	X	___	___	___	___	Awful
Slow	___	___	___	___	___	___	X	Fast
Sour	___	X	___	___	___	___	___	Sweet
Alive	___	___	X	___	___	___	___	Dead
Good	___	___	X	___	___	___	___	Bad
Quiet	___	___	___	___	___	___	X	Noisy

Figure 4.3 Instructional Program Provided For Karen from April 6, 1970 to June 26, 1970

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignments	Results
April 6-10	5	writing addition sentences using words and numerals commutative and associative properties of union addition facts with sums less than or equal to nine commutative property of addition	poker chips and paper-and-pencil; first teacher manipulated and Karen wrote, then teacher wrote and Karen manipulated--Karen also wrote sentences using words and numerals made flashcards of addition facts with sums less than or equal to eighteen to develop speed and accuracy--worked on facts less than ten 100 addition facts on timed test (21/30 of 100 in two minutes and 49/96 of 100 in four minutes best score during week) began making flash cards of subtraction facts with minuends less than or equal to eighteen	textbook involving the use of the number line in demonstrating addition sentences such as 5 + 2 = 7 and 3 + 4 = 7 worksheets involving union of sets which demonstrated the commutative property of addition 100 addition facts on timed test 20/100 in four minutes worksheets involving addition facts less than ten	Karen had difficulty writing out her mathematics sentences in words. She had more success in writing appropriate number sentences. She consistently spelled words phonetically rather than as they were supposed to be; e.g., "right" as "ate". Karen needed more help with the commutative property of union. She realized that joining sets of objects in different ways did not change the results, but she still became confused when the order of the sets were changed and then joined. Karen had difficulty adding without counting. She was able to add combinations involving predominantly 1, 2, and 3 but had to count more complex combinations using the numberline and fingers as crutches.
April 13-17	0	none (spring vacation)	none	none	none
April 20-24	5	joining or union of elementally grouped arrays like x x / x x x and x x x x identity element of addition addition facts with sums less than or equal to eighteen commutative property of addition subtraction facts with minuends less than ten regrouping ones to tens	shown elementally grouped arrays of dots, 1-9, where there are not more than four dots in a row--was asked to make up her own number sentences joining the two sets together flash cards with addition sums less than or equal to eighteen--emphasis placed on the identity element poker chips used to demonstrate commutative property of addition as teacher wrote number sentences on the chalkboard teacher used chalkboard to demonstrate regrouping of ones to tens example of one exercise for addition used: Show: 8 + 2 = □, "Start with eight and count up two."	textbook assignments involving problems like 6 + 4 = 10 and 10 + 0 = 10 worksheet assignments involving problems requiring Karen to fill in addend, like 4 + 1 = 5 and 1 + 1 + 4 = 7, thus allowing her to make up her own problems timed daily two minute writes using 100 addition facts worksheets of story problems involving addition and subtraction--asked to make up own using physical objects	Karen got the identity element of addition mixed up with the adding "one more than property" of one. Karen appeared to enjoy making up her own addition problems when she was required to fill in blanks leading to a given sum. When asked to make up story problems using physical objects she understood what she was doing. She needed more help. Karen made her own flash cards and practiced with Cinthia. She still needed lots of help.

Figure 4.3 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignments	Results
April 27 - May 1	5	addition facts with sums less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen regrouping ones to tens addition and subtraction as inverse processes place value	addition and subtraction flash cards with sums and minuends less than or equal to eighteen to build speed and accuracy worksheets of addition and subtraction problems as inverse processes, like 7 + 2 = ___ and 9 - 2 = ___; also $\begin{array}{r} 5 \\ +3 \end{array}$ & $\begin{array}{r} 8 \\ -3 \end{array}$ used poker chips to demonstrate that subtraction is the undoing of addition Cuisenaire blocks used to demonstrate simple addition situations where regrouping of ones to threes, ones to sevens, and then ones to tens was necessary	story problems involving simple addition and subtraction timed daily two-four minute writes using 100 addition and 100 subtraction facts worksheets using the counting sam, demonstrating the regrouping of tens and ones, like 8 tens and 6 ones = 80 + 6 = 86 bundles of pencils representing tens and ones--problems like 70 + 6 = 76, 10 + 7 = 17, and 29 = 20 + 9	Identity element of addition was mastered. Karen still needed answering without counting with addition facts when sums were larger than 9 or equal to or greater than 10. Karen voiced understanding of the commutative property of addition, but when put in the position where she needed to quickly think of it she could not. Subtraction facts with minuends less than 10 was not mastered. There was no apparent link between addition and subtraction as inverse processes. Therefore, except for 1, 2, and 3 being subtracted, rote counting on fingers and numberline were the procedures followed to obtain answers. Karen did not master regrouping ones to tens. She performed the acts and was unable to specify what she had done. When shown a number of problems like 8 + 2 = ☐ Karen was able to orally state the problem and prove her work by manipulating poker chips. However, the proof was by counting one chip at a time. Addition facts with sums less than or equal to 18 was not mastered. Karen had difficulty with minuends greater than 9 and differences greater than or equal to 5. Karen seemed to understand the regrouping process when the algorithm was aligned vertically but had difficulty with the process when in story form or horizontally aligned. The use of the Cuisenaire materials momentarily clarified the need to regroup. More work was needed in regrouping ones to tens. Karen seemed to master addition and subtraction as inverse processes. Karen habitually is inconsistent from one day to the next. Mastery of an idea one day did not mean that she retained the idea the next day. However, over a two

Figure 4.3 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Resources Assignment	Results
May 4-8	5	addition facts with sums less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen regrouping ones to tens for addition regrouping tens to ones for subtraction	addition and subtraction flash cards with sums and minuends less than or equal to eighteen counting man emphasizing regrouping tens to ones and ones to tens when given related addition and subtraction number sentences abacus emphasizing regrouping and place value, given appropriate verbal and written number sentences	worksheets emphasizing simple subtraction problems like 91 & 70 -7 -41 timed worksheets involving addition (two minute writes 11/31 of 31/36 addition facts less than ten best performance) (three minute writes 45/56 addition facts less than ten best performance)	day period when she expressed an idea with understanding the investigator reported the task as "seemed to master". Karen was very close to mastering addition facts less than ten. When placed in the stress situation of a timed test at times she got flustered and did not perform well. Karen needed more help on subtraction facts with minuends less than or equal to eighteen. She could quickly think of the subtraction facts with differences less than 4 but resorted to counting on more complicated problems. Karen needed more drill and practice in various forms on regrouping ones to tens and tens to ones in addition and subtraction respectively. There were indications that she was getting the ideas of regrouping.
May 11-15	4	addition facts with sums greater than nine and less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen regrouping ones to tens for addition regrouping tens to ones for subtraction	same as May 4-8 addition and subtraction worksheets requiring Karen to regroup were passed out daily bingo using addition and subtraction number facts	worksheets using counting man demonstrating problems like 35 - 3 = 30 + □. worksheets with problems like 52 & 27 +4 +3 worksheets using , , and =, like 32 + 4 30 + 7 worksheets with problems like 36 + 3 = 30 + □ = ___ and 30 + 3 / 4 / 33 + 7 = 37	Karen needed more help with addition facts greater than nine and less than or equal to eighteen. At times Karen reverted back to not being able to identify addition facts less than ten. Karen was still having extreme difficult in working out subtraction facts without consulting the numberline or counting on her fingers. Regrouping of ones to tens still was not mastered. It appeared that she did not see the need to regroup because of an overload in one column. She was able to answer problems in vertical algorithm form. At times she gave the impression that she did not understand the link between notation and that the problem could be demonstrated using physical objects. Regrouping tens to ones was not understood as well as regrouping ones to tens. Karen understood what <, >, and = meant.

Figure 4.3 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
					At times it appeared that Karen understood what she was doing using vertical and horizontal forms to compute addition problems, however, she continually got mixed up and made wild guesses at answers. She found it necessary to count out each answer and had to start all over with a problem when interrupted; she could not pick up her work where she left off.
May 18-22	5	addition facts with sums less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen commutative property of addition	addition and subtraction flash cards with sums and minuends less than or equal to eighteen bingo using addition and subtraction number facts 10 x 10 matrix of addition facts to show commutative property	same as May 11-15	Karen needed more help on her addition and subtraction facts. However, she enjoyed playing bingo and was excited for the special arithmetic sessions to come. She voiced disapproval about not having the special sessions daily. Regrouping from ones to tens and from tens to ones continued to present the same perplexities as described during the week May 11-15.
May 25-29	3	addition facts with sums less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen regrouping ones to tens for addition addition and subtraction as inverse processes regrouping tens to ones for subtraction	bingo using addition and subtraction number facts addition and subtraction flash cards with sums and minuends less than or equal to eighteen sets of blocks, pencils, poker chips, etc.--teacher demonstrated addition and subtraction as inverse processes, child wrote on chalkboard--teacher wrote number sentences demonstrating addition and subtraction as inverse processes on chalkboard and child manipulated objects	counting sam used to demonstrate necessity of regrouping in addition and subtraction--three pronged abacus also used timed two minute test of addition facts with sums greater than nine and less than or equal to eighteen worksheets with problems like $48 - 2 = 40 + \square$ and column addition with up to four addends and up to three digits	Addition of facts with sums less than or equal to 10 were mastered. Karen still needed help on addition facts with sums greater than ten. Addition and subtraction as inverse processes again demonstrated as being mastered. Regrouping ones to tens seemed to be mastered. However, more drill and practice was needed to concretionalize the idea. Subtraction facts with minuends less than 10 were nearly mastered. Karen had difficulty conceptualizing what regrouping tens to ones for subtraction meant. She was able to use the rote method of borrowing, however and carefully went through step-by-step procedures to arrive at her answers..

Figure 4.3 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignments	Results
June 1-5	2	addition facts with sums less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen regrouping ones to tens for addition regrouping tens to ones for subtraction commutative and associative properties of addition as is related to subtraction	bingo using addition and subtraction number facts worksheets using the counting man as a model for teaching regrouping ones to tens for addition and regrouping tens to ones for subtraction using stacks of poker chips the teacher demonstrated why the commutative and associative properties of addition are not applicable for subtraction	same as May 25-29 worksheet problems accentuated subtracting a number from itself	Karen had not mastered addition facts greater than ten. When the answers got into larger numbers she seemed, at times, to guess at the answers. She was trying and improving, especially during the bingo games. Karen mastered subtraction facts with minuends less than ten but her answers with differences of numbers greater than 6 were at times seemingly at random. Regrouping ones to tens needed more work. Regrouping tens to ones needed more work. Karen was not allowed to respond to the commutative and associative properties of addition as is related to subtraction. There was also no indication of mastery.
June 8-12	2	same as June 1-5 commutative property of addition setting up addition and subtraction story problems	bingo using addition and subtraction number facts addition and subtraction flash cards with sums and minuends less than or equal to eighteen worksheets using the counting man as a model for teaching regrouping for addition and subtraction	worksheets of addition and subtraction story problems drill and practice of written addition problems vertically and horizontally aligned--the problems were sequentially presented from simple to complex, requiring a more indepth understanding of needed skills to compute the more complex problems	Karen still had difficulty with addition facts with sums greater than nine but less than eighteen. She knew the facts with sums less than ten. Karen still had difficulty with commutative property of addition but mastery was evidenced with the facts having sums of less than ten. Karen continued to have difficulty with regrouping tens to ones but mastered regrouping ones to tens, tens to hundreds, etc. Subtracting a number from itself was mastered. Karen was successful in answering story problems requiring the use of addition. She could read the problems involving subtraction but her work was not consistent. At times she knew what she was supposed to do but at other times her responses were not logical.

Figure 4.3 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homework Assignments	Results
June 15-19	2	addition facts with sums less than or equal to eighteen subtraction facts with sums less than or equal to eighteen regrouping tens to ones for subtraction subtracting zero from any whole number	same as June 8-12	worksheets of addition and subtraction story problems drill and practice of written subtraction problems vertically and horizontally aligned--the problems were sequentially presented from simple to more complex, requiring a more indepth understanding of needed skills to compute the more complex problems	Karen continued having difficulties with addition facts with sums greater than nine but less than eighteen. Karen mastered the commutative property of addition. Karen continued doing subtraction problems requiring regrouping in a rote manner. When questioned about what she had done she could not explain her procedure. Karen mastered subtracting zero from any number.
June 22-26	2	review number facts in addition and subtraction take achievement test	100 basic addition facts on timed test 100 basic subtraction facts on timed test bingo using addition and subtraction number facts	worksheets of addition and subtraction story problems	See post-test data in Table 4.1.

Karen demonstrated mastery of many addition and subtraction skills at the conclusion of the study that she could not perform with mastery at the beginning of the study. Whereas Karen had not mastered tasks such as "write meaningful symbols for 0-9", "commutative property of union", "identity element of addition", "addition and subtraction as inverse processes", and higher-level tasks in the hierarchies in Appendix A, except "ability to read problems", many of these tasks and others were mastered by the conclusion of the study. For instance, Karen demonstrated mastery of all the tasks in the addition hierarchy except "addition facts with sums greater than nine but less than eighteen", "compute by regrouping (carrying) and non-regrouping", and "setting up and solving word problems involving addition". Karen demonstrated mastery of all tasks in the subtraction hierarchy except for "subtraction facts with minuends greater than nine but less than or equal to eighteen", "compute by grouping tens to ones", "compute by regrouping (borrowing) and nonregrouping", and "setting up and solving word problems and had difficulty setting up and solving subtraction problems. The above mentioned skills are reported in the post-assessment of arithmetic tasks in Table 4.1.

Throughout the instructional program, there were obvious indications that Karen tired of many of the drill and practice activities provided for her. The sequential arrangement of tasks was not adhered to as consistently as suggested in the instructional design. Also, tasks were not fully mastered before moving on to higher-order tasks, perhaps inhibiting learning rather than enhancing it. Furthermore, there were obvious indications that Teacher A made an attempt to teach both Karen and Cinthia with the same or similar instructional plans, rather than individualizing the program specifically for Karen's needs.

At the conclusion of the study, Karen responded to the attitude scale in a mediocre manner, more positive than negative, however. Whereas she had to be coerced into participation at the beginning of the study Karen continued to voice disapproval when she could not have her special sessions because of disruptive schedules, etc. Karen's homeroom teacher mentioned that during the time the study was conducted Karen had shown more improvement in arithmetic than she had previously and that her attitude had continued to improve as she became more and more successful in demonstrating why mathematical actions work.

Case Study of Cinthia

Biographical Sketch

Cinthia Britton entered Edgefield Lodge one month prior to the beginning of this study. She was described by the staff at the school as an alert, attractive ten-year-old with many deceptive and manipulating characteristics. She was noted for setting up both children and adults in combative situations and stepping aside to watch the results. This was apparent at home and at school. Some of the adjectives used to describe Cinthia's behavior were: deceptive, colossal manipulator, overindulged, destructive, violent with other kids, hurts smaller children, possessive, aggressive, perceptive, conniving, diabetic, intimidating, plays the incompetent role, very verbal, takes over classroom discussion, physically coordinated, etc.

Cinthia was an only child of her adoptive parents, Mr. and Mrs. Theodore Britton, who adopted Cinthia when she was an infant. When Cinthia was two years old Mrs. Britton passed away. Until Cinthia was five years old she was raised by a series of housekeepers and an over-indulgent grandmother. In 1965 Mr. Britton married the present Mrs. Britton, who had two teenage sons. Cinthia was a major source of conflict between Mr. and Mrs. Britton from the beginning of their marriage. Being a diabetic she used her condition to manipulate them, disallowing any privacy for them and on numerous occasions creating friction between them.

Cinthia was supposed to have been in the fourth grade but had been retained one year in the first grade. At the time both her teacher and her parents felt that it was wise to retain her in order for her to be successful in school. Cinthia used her situation to her advantage by manipulating the younger and smaller children in her classroom and she soon became known as a trouble maker. For approximately a year prior to her entrance to Edgefield Lodge Cinthia had only been allowed to attend school on a part-time basis. Just prior to her entrance to Edgefield Lodge her behavior had become so unmanageable that she was suspended from school and allowed to return to her classroom for a short time for observational purposes only. She had difficulty remaining at-task long enough to make the experience worthwhile for everyone concerned. Cinthia was a constant threat to her teacher because she defied everything the teacher tried. Her teacher made it a point to request that Cinthia be removed from her room because of the child's anti-social behavior.

Cinthia was working at a third grade level in everything but mathematics, which was at a 2.5 grade level of performance. She was able to read fluently but had difficulty comprehending what she read. Cinthia did everything she could to get out of work, including purposefully losing her assignments, destroying books, sluffing school, playing sick, etc. She verbalized nicely, but when placed in problem solving situations she resorted to disruptive and rude behavior.

Diagnostic-Manipulative Arithmetic Program

Preassessment of Arithmetic Competencies. Cinthia was performing at a 2.5 grade level in arithmetic and she had many problems with addition and subtraction concepts. The results from the revised researcher-designed diagnostic-achievement test, given prior to the beginning of the study and reported in Table 4.1, showed that she had mastered tasks leading up to an understanding of the "principle of addition", except for "associative property of union". However, Cinthia had not mastered higher-level addition tasks, except for "ability to read Problem". Regarding subtraction, Cinthia had mastered "set separation", "principle of subtraction", and "addition and subtraction as inverse processes" but had not mastered higher level tasks, except for "ability to read problem".

Cinthia thought highly of her ability to perform arithmetic tasks. However, when asked to explain the reason behind the mathematical actions she declined. There was every indication that Cinthia performed arithmetic at a rote, rule oriented level, and had developed few arithmetic skills, including a knowledge of basic addition and subtraction facts.

Instructional Program. Instruction began for Cinthia on April 6, 1970. Teacher A was her teacher for the special arithmetic sessions, providing thirty-five instructional periods over twelve weeks. Cinthia and another child, Karen received instruction together and seemed to work cooperatively.

Figure 4.1 is a descriptive account of the arithmetic program provided for Cinthia during the instructional phase of the study. Also, Cinthia's attitude toward her ability to do arithmetic is tested by her response on the semantic differential attitude scale in Scale 4.4. She was asked to respond to the attitude scale on the final day of special arithmetic instruction, June 26, 1970. Furthermore, post-instructional arithmetic skills are reported in Table 4.4. The instructional program and Cinthia's attitude changes are analyzed at the conclusion of the case study.

Scale 4.4: Cinthia's Attitude Toward Her Ability To Do Arithmetic Following Instruction

Big	___	X	___	___	___	___	___	Little
Unhelpful	___	___	___	___	___	___	X	Helpful
Old	___	___	___	___	___	___	X	Young
Strong	___	X	___	___	___	___	___	Weak
Powerless	___	___	___	___	X	___	___	Powerful
Deep	___	___	___	___	___	___	X	Shallow
Nice	X	___	___	___	___	___	___	Awful
Slow	X	___	___	___	___	___	___	Fast
Sour	___	___	___	X	___	___	___	Sweet
Alive	X	___	___	___	___	___	___	Dead
Good	___	___	___	X	___	___	___	Bad
Quiet	X	___	___	___	___	___	___	Noisy

Figure 4.4: Instructional Program Provided for Cinthia from April 6, 1970 to June 24, 1970

Week	No. of Days	Tasks:	Materials and Activities	Homework Assignments	Results
April 6-10	5	writing addition sentences using words and numerals commutative and associative properties of union addition facts with sums less than or equal to nine commutative property of addition	poker chips and paper-and-pencil; first teacher manipulated and Cinthia wrote, then teacher wrote and Cinthia manipulated--Cinthia wrote sentences using both words and numerals made flash cards of addition facts less than or equal to eighteen to develop speed and accuracy--worked on facts less than ten 100 addition facts on timed tests (21/21 of 100 in two minutes and 84/84 in four and one-half minutes best score during week) began making flash cards of subtraction facts	worksheet with 100 addition facts drill and practice of written addition problems vertically and horizontally aligned--the problems were sequentially presented from simple to complex worksheets with simple addition and subtraction story problems	Cinthia at first had difficulty writing her own sentences using words and numbers. She later accepted it as a challenge and as the week ended she reacted to this activity as if it were a game. At times her spelling was not accurate but she finally realized that number sentences are merely short ways of putting oral and written word sentences. Cinthia mastered the commutative and associated properties of addition. Cinthia needed a lot more help with addition facts. Most of the time she counted answers out on her fingers. When working on computational exercises she became frustrated and wrote numerals down at random, apparently just to get finished. Cinthia had a lot of trouble with horizontally aligned problems. Cinthia voiced that she liked to do math but she quickly became disinterested in continued drill and practice and asked to move on to more difficult things. Cinthia read the story problems asked of her but seemingly put answers down at random.
April 13-17 (spring vacation)	0	none	none	none	none
April 20-24	5	addition facts with sums less than eighteen commutative property of addition subtraction facts with minuends less than ten regrouping ones to tens	addition flash cards with sums less than or equal to eighteen --emphasis was placed on the identity element which Cinthia mastered during the week finished making flash cards of subtraction facts with minuends less than or equal to eighteen poker chips used to demonstrate commutative property of addition as teacher wrote number sentences on the chalkboard; vise versa	same as April 6-10 worksheets with counting men demonstrating regrouping ones and tens	Cinthia still had to count to find answers for basic arithmetic facts. She answered addition facts quickly and correctly when the minuend was less than six. Cinthia counted out answers to higher order number facts. Cinthia mastered regrouping ones to tens.

Figure 4.4 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homework Assignment	Results
			teacher used a chalkboard example to demonstrate regrouping ones to tens example of one exercise for addition used: show 8 + 2 = □, "Start with eight and count up two."		
April 27 - May 1	5	addition facts with sums less than eighteen subtraction facts with minuends less than eighteen addition and subtraction as inverse processes regrouping ones to tens regrouping tens to ones	addition and subtraction flash cards with sums and minuends less than or equal to eighteen to build speed and accuracy addition timed tests of facts with minuends less than ten subtraction timed tests of facts with minuends less than six worksheets of addition and subtraction problems as inverse processes, like 7 + 2 = ____ and 9 - 2 = ____; also 5 & 8 +3 & -3 used poker chips to demonstrate that subtraction is the undoing of addition cuisenaire blocks used to demonstrate simple addition situations where regrouping of ones to threes, ones to sevens, and then ones to tens was necessary	worksheets with counting man demonstrating regrouping ones and tens worksheets of two and three digit addition problems requiring regrouping (carrying) worksheet of two digit subtraction problems requiring regrouping (borrowing) daily two minute writes on addition facts daily two minute writes on subtraction facts worksheets of story problems involving simple addition and subtraction	Cinthia was close to mastery of addition facts with sums less than ten. The only one combination that she did not know without counting was 6 + 3 and 3 + 6. Cinthia had difficulty with subtraction facts with minuends less than ten when the differences were three or greater. Cinthia mastered the idea that addition and subtraction are inverse processes. Cinthia had difficulty explaining regrouping ones to tens and tens to ones. She was able to visualize what was happening when the teacher used the chalkboard in explaining base ten addition and subtraction and when the Cuisenaire rods were used, but she got bogged down when she was requested to explain the process using vertically aligned problems. She reverted to using hash marks and a rote explanation of procedures. Cinthia's timed addition and subtraction fact tests continued to improve. However there were days that she regressed.
May 4-8	5	addition facts with sums less than or equal to eighteen subtraction facts with minuend less than or equal to eighteen regrouping ones to tens regrouping tens to ones place value	addition and subtraction flash cards with sums and minuends less than or equal to eighteen to build speed and accuracy worksheets of addition and subtraction problems as inverse processes like 7 + 2 = ____ and 9 - 2 = ____; also, 5 & 8 +3 & -3 used poker chips to demonstrate	worksheets with problems like 80 + 7 and 40 59 -(10 + 4) +18 -18 etc. 60 + 11 = 71 worksheet with 100 addition facts, 2 minutes---33/37 worksheet with 100 subtraction facts, 4½ minutes---84/96	

Figure 4.4 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
			used Cuisenaire rods to demonstrate place value used seven pronged abacus to demonstrate place value	worksheet differentiating operation and equivalent signs; i.e., +, -, <, >, = like, 37 + 12 = 49 and 26 + 23 = 27 + 22 worksheet with horizontally aligned addition and subtraction problem; i.e., 4 + 2 = 6, 6 - 4 = 2, etc. worksheets of story problems involving simple addition and subtraction	Cinthia needed more help on place value since she had not worked with many numbers as large as a hundred, thousand, etc. She did know what was being mentioned about over leading digits in base three, seven, and ten using Cuisenaire rods. Cinthia mastered regrouping of ones to tens for addition but continued to have difficulty with regrouping tens to ones for subtraction. Cinthia continued to have problems with the operation signs in story problems.
May 11-15	3	addition facts with sums less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen regrouping tens to ones for subtraction compute by regrouping and non-regrouping for addition and subtraction	addition and subtraction flash cards with sums and minuends less than or equal to eighteen counting man emphasizing regrouping tens to ones and ones to tens when given related addition and subtraction number sentences abacus emphasizing regrouping and place value, given appropriate verbal and written number sentences addition and subtraction worksheets requiring Cinthia to regroup were passed out daily bingo using addition and subtraction number facts	same as May 4-8	Cinthia had mastered addition facts up to 7 + 7. At times she got 6 + 7 and 7 + 6 mixed up with 7 + 5 and 5 + 7, however, she continued to try and became upset with herself when she missed a problem because of a mistake that she had corrected previously. Cinthia continued having difficulty with higher ordered subtraction facts. She had not gotten the associative and regrouping ideas like 9 + 6 = (9 + 1) + 5 = 15. She had to count out her answers and that slowed her computational workdown. Cinthia continued having problems with place value. However, at times she asked questions like "why isn't there any more numbers than 1-9?" She forgot about zero and had a need for an answer but did not realize why numbers move into two digit numbers after 9. Cinthia demonstrated the lack of need to do many of her addition problems with two or more digits without hash marks as a crutch. However, subtraction problems requiring regrouping were still being done in a rote fashion. Cinthia expressed excitement about playing bingo with number facts.

Figure 4.4 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignments	Results
May 18-22	2	addition facts with sums less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen commutative property of addition	addition and subtraction facts on flash cards with sums and minuends less than or equal to eighteen bingo using addition and subtraction number facts 10 x 10 matrix of addition facts to show commutative property	textbook exercises demonstrating that a set of objects is "n" ones or one set of "n" objects textbook exercises demonstrating that bundles of pencils equals ten and singel ones, therefore three bundles of pencils and three ones equals thirty-three, etc. textbook exercises demonstrating that arrays of ten stars, dots, diamonds and triangles equals ten--also, asked Cinthia to specify what the sets stood for by writing out in long hand problems like "6 tens and 7 ones" textbook exercises using dimes and pennies to demonstrate place value	Cinthia mastered addition facts with sums less than or equal to eighteen. She almost mastered subtraction facts with minuends less than ten. She got stumped on $\begin{array}{r} 9 \\ -1 \end{array}$ & $\begin{array}{r} 8 \\ -5 \end{array}$ Cinthia filled in the 10 x 10 matrix without hesitation. Cinthia had difficulty with place value, missing a large number of the problems on the textbook exercises.
May 25-29	3	addition facts with sums less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen	addition and subtraction facts on flash cards with sums and minuends less than or equal to eighteen bingo using addition and subtraction number facts	100 addition facts on timed test 100 subtraction facts on timed test textbook exercises with arrays of dots signifying tens and ten sets of ten or one hundred - also, bundles of ten pencils and single pencils for two digit numbers textbook exercises, boxes of ten bundles of pencils, bundles of ten pencils and single pencils for three digit numbers--also, stacks of boxes for thousands textbook exercises using a rod of ten unit cubes, a flat of ten unit cubes, and a cube with ten X ten X ten unit cubes textbook exercises with review of place value problems; e.g., 6 th, 5 h, 2 t, and 4 = _____; which number has more hundreds 6325 or 6425; give the number that the underlined digit represents - 6720; 6721 means ___hundreds, 6 thousands, 1 one, 2 tens	Cinthia mastered subtraction facts with minuends less than ten. She continued having problems with subtraction facts with minuends greater than nine and less than or equal to eighteen. Cinthia again displayed mastery of addition facts with sums less than or equal to eighteen. Cinthia still seemed confused about place value with digits greater than the tens column

Figure 4.6 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homework Assignment	Results
June 1-5	3	addition facts with sums less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen regrouping ones to tens in addition regrouping tens to ones in subtraction commutative and associative properties of addition as related to subtraction	bingo using addition and subtraction number facts addition and subtraction flash cards with sums and minuends less than or equal to eighteen worksheets using the counting men as a model for teaching regrouping ones to tens for addition and regrouping tens to ones for subtraction using stacks of poker chips the teacher demonstrated why the commutative and associative properties of addition are not applicable for subtraction	textbook exercises requiring Cinthia to use "less than" and "greater than" symbols properly for numbers with up to three digits textbook exercises requiring the identification of place value up to seven digits textbook exercises requiring the use of an addometer	Cinthia continued having trouble with subtraction facts with minuends greater than nine and less than eighteen. Cinthia had mastered regrouping in addition but still was confused about regrouping in subtraction. Commutative and associative property of addition as is related to subtraction was mastered by Cinthia. Cinthia was very much aware of "less than" and "greater than" relationships up to four digit numbers. Cinthia became confused with numbers in the ten thousands and above. Cinthia figured out the problems using the adometer.
June 8-12	1	addition facts with sums less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen	bingo using addition and subtraction number facts worksheets using the counting men as a model for teaching regrouping for addition and subtraction	textbook exercises of addition and subtraction on the numberline	Cinthia continued having trouble with subtraction facts with minuends greater than nine and less than eighteen. Cinthia was absent from school all but one day of this week.
June 15-19	2	same as June 8-12 subtracting zero from any whole number	same as June 8 - 12 addition and subtraction flash cards with sums and minuends less than or equal to eighteen	drill and practice of written addition and subtraction problems vertically and horizontally aligned in the problems were sequentially presented from simple to complex, requiring a more in depth understanding of needed skills to compute the more complex problems	Cinthia was still having problems with subtraction facts having differences of 7, 8, and 9. Cinthia made a few mistakes on regrouping subtraction problems. Cinthia had mastered computing addition problems requiring regrouping and non-regrouping.
June 22-26	2	review number facts in addition and subtraction take achievement test	100 basic addition facts on timed test 100 basic subtraction facts on timed test bingo using addition and subtraction number facts	worksheets of addition and subtraction story problems	See post-test data in Table 4.1.

According to the post results on the researcher-designed diagnostic-achievement test Cinthia had mastered all of the tasks identified in the addition hierarchy in Appendix A, except "setting up and solving word problems involving addition". With the final task Cinthia was not able to link the setting up portions with the solving of the arithmetic problems.

Cinthia found the operation of subtraction more difficult than addition although she made substantial gains with subtraction also. She was unable to master the tasks "subtraction facts with minuends greater than nine and less than or equal to eighteen", "subtracting a number from itself", "compute by regrouping tens to ones", and higher level tasks. Cinthia became overjoyed when she discovered a pattern or relationship and on numerous occasions approached the researcher to tell him about it.

Instruction provided for Cinthia did not follow the instructional design as closely as was specified in Chapter III. For instance, tasks were often presented out of sequence and at times tasks were omitted. Perhaps because of this Cinthia often became frustrated. Some manipulative and active learning exercises were introduced and Cinthia seemed to benefit from them.

At the conclusion of the study Cinthia responded in a positive manner to the bi-polar items on the attitude scale reported in Scale 4.4. Throughout the duration of the study Cinthia made it a point to seek out the investigator and mention some of the fun things she was involved with in her special arithmetic sessions. Since Cinthia was new to the school there was very little to compare with regarding attitude change in the classroom, however, her classroom teacher did mention that she was willing to attempt more difficult material in arithmetic as a result of the arithmetic program. Whereas Cinthia sat idle and was unable to do many of the assignments presented her when she first came to the school, by the conclusion of the study she would attempt to do the assignments without the constant help of the teacher. She had developed a willingness to try difficult arithmetic tasks rather than elicit the aid of anyone who was willing to help.

Case Study of Michael

Biographical Sketch

Michael Burke was a small eight year old whose slow-moving, uncooperative modes of behavior resulted in his placement in Edgefield Lodge. Michael had learned to respond to adult requests in a catatonic manner; i.e., acting as if he had not heard the request and just staring off into space in a world of fantasy. At times he acted the role of an incompetent mentally retarded child; however, his behavior at other times indicated that he could perform as any normal child might. Some of the adjectives used to describe his behavior were: restless, fearful, withdrawn, catatonic, immature, hostile, potentially normal, engaging in attention-manding and giving up behaviors, non-verbal, nonresponding, competence-anxiety, mand-avoidant, stubborn, hyperactive, infantile behavior, coordinated, difficulty warming up to people, low vocabulary, stalling, etc. After Michael's admittance to the school, he gradually got the idea that his ideas were to be shared and later was described as "jabbering like a jaybird." However, he returned to his quiet, avoidance manner when placed in situations where he was to interact with adults who were strange to him.

Michael and his younger brother, Jeff, had spent over half their lives in foster homes. Their two older brother and older sister lived at home with their parents. Mrs. Burke was described as a psychotic, having a history of mental disturbances and periods of confinement in mental institutions. Mr. Burke was described as never having held a permanent job, perhaps because he was intellectually slow. However, indications were that Mr. Burke had been the one to hold the family together. The Burkes were perpetual welfare recipients. Because of continued mistreatment and neglect, Michael and Jeff had been permanently removed from the home by the juvenile court.

Michael had little difficulty interacting with children, especially when the children were his own age. When confronted with adults who were strangers or who expressed strong authoritative demands Michael would withdraw and would behave as an infantile mentally retarded child might. However, by the time this study was initiated Michael had learned to respond to adults who expressed kindness to him. In fact, his behavior had changed from being overly reserved to a hyperactive way of reacting to situations of excitement. Michael was taken away from the foster parents who referred him to the school and was placed in a home of a young couple with boys his own age. Mrs. Schultz, Michael's new foster mother, had expressed concern about how to help him accept new and fun situations without becoming overly excited.

When Michael entered Edgefield Lodge he was listed as a seven-and-one half year old kindergartener. Michael had progressed to where he was working at a mid-first grade level in everything except mathematics.

From the school progress reports prior to the time of this study, mathematics was listed as one of Michael's least liked subjects and he had adopted various crutches to help himself, such as making hash marks or counting on a numberline to identify addition facts.

Diagnostic-Manipulative Arithmetic Program

Preassessment of Arithmetic Skills. Even though it was reported that Michael was performing at a first grade level in arithmetic, much of his performance was at a rote, rule-oriented level of performance. The pre-test of the revised researcher-designed diagnostic-achievement test, reported in Table 4.1, indicated that Michael had little competency in addition and subtraction of whole numbers. Michael had not mastered tasks beyond "discriminate and classify characteristics logical to all objects in a set", as indicated from the hierarchies in Appendix A. He was able to describe characteristics of sets in his own limited vocabulary.

Instructional Program. Instruction began for Michael on April 3, 1970. Teacher A was his teacher for the special arithmetic sessions, providing thirty-eight instructional periods over twelve weeks. Michael and another child, Robert, received instruction together until May 18-22 when Robert was dropped from further investigation because of continued illness. Also, during two weeks of the instructional phase Michael was provided instruction with Karen and Cinthia, however, he was not able to compete in some of the activities and was taught separately from that point to the end of the study.

Figure 4.1 is a descriptive account of the arithmetic program provided for Michael during the instructional phase of the study. Also, Michael's attitude toward his ability to do arithmetic is tested by his responses on the semantic differential attitude scale in Scale 4.5. He was asked to respond to the attitude scale on the final day of special arithmetic instruction, June 26, 1970. Furthermore, post-instructional arithmetic skills are reported in Table 4.5. The instructional program and Michael's attitude changes are analyzed at the conclusion of the case study.

Scale 4.5: Michael's Attitude Toward His Ability To Do Arithmetic Following Instruction

Big	X							Little
Unhelpful				X				Helpful
Old	X							Young
Strong				X				Weak
Powerless				X				Powerful
Deep				X				Shallow
Nice				X				Awful
Slow	X							Fast
Sour				X				Sweet
Alive				X				Dead
Good	X							Bad
Quiet							X	Noisy

Figure 4.8 Instructional Program Provided for Michael
From April 3, 1970 to June 26, 1970

Week	No. of Days	Tasks	Materials and Activities	Resource Assignment	Results
April 3	1	discriminate and classify characteristics to all objects in a set discriminate and classify differences in a set of objects	HELP items 2-5: (see Appendix C) color identification (ten colors) bead designs (sequencing patterns) block design (shape patterns) bolt board (size variation)	worked with materials in HELP kit in a non-directed manner	Michael was able to identify the names of all the primary colors (red, yellow, blue), secondary colors (orange, purple, green), brown, pink, and grey. Michael was able to duplicate bead patterns. However, on more than one occasion he reversed the patterns but later changed it. He was not able to string the bead patterns by memory nor did he create his own design. Michael had difficulty working with the bolt board (wooden strand, nine holes of diminishing sizes fitted with bolts and nuts to match). He was able to take apart, mix, and assemble the bolts and nuts on the bolt board, however, he was not able to explain the principles he used. Michael was able to arrange nine colored blocks to match patterns of similar size on cards to the blocks; however, he was not able to use the same block to match miniaturized patterns on cards. He became easily distracted. Michael became tired of an activity easily. After seven or eight minutes on exercises he would not remain at-task. Also, Michael's habit of using as few words as possible when speaking with adults created a problem in determining his logical thinking in performing the tasks. There were indications that many of the questions and comments made were misinterpreted because of vocabulary. Prior to the testing session Michael had been allowed to work with the items in the HELP kit without any teacher help.
April 6-10	4	same as April 3 recognition of sets	HELP items 7, 9-11 (see Appendix C) numbers (more, less, many, same etc.); writing a name (letter configurations, etc.); auditory perception (name of small toys, sorted, renaming sets, etc.); social interaction (identified rules, agreements, or standards in conflict situations)	worksheets requiring matching one-to-one to determine if sets had the same number--was required to write short sentences describing which set had less and which set had more worked on dot-to-dot puzzles using numerals 1-30 worked on fitting in dates on calendars	Michael was able to place the plastic pieces of different lengths and colors in seriated order according to length. There was some question whether he attached the concept of order to the length or to the numeral written on the plastic pieces. Each plastic piece had the appropriate number of outlined squares as indicated by the accompanied numeral and length of object. Michael was able to count to ten but did not know what

Figure 4.3 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
			paper and crayons-drew squares, circles, and triangles of different colors--talked constantly about color relationships	worksheets telling time	was "two less than six", for instance.
			every piece of paper had full name in upper and lower case print; date was also written on each paper		Michael's printing was immaculate. He took great care in writing his name. All of the letters were appropriately written in upper and lower case. He was slow in printing anything.
			bead design-emphasized stringing beads in patterns of shape and color from memory		Michael had difficulty discriminating "p and b" and "d and t" as consonant sounds. Perhaps that was because he spoke using the same speech inaccuracies. He was able to identify names of objects with similar initial consonants.
			belt board--emphasized taking belts off board, mixing them, then putting them back--was encouraged to organize them in seristed order according to width and length.		Michael was not very social conscious. He verbalized poorly and as a result had difficulty explaining this section of the test. However, he was aware of rules and standards and explained many of the rules established in his homeroom class.
					Michael had little difficulty on one-to-one matching and the determination of what was meant by the words more, less, and same. He had difficulty with the vocabulary, but once that was cleared up he learned quickly.
					Michael had little difficulty with dot-to-dot puzzles and seemed to understand the patterns involved in going from 1-10.
					Michael worked with calendars and clocks with apparent understanding.
April 13-17	0	none (spring vacation)	none	none	none
April 20-24	5	same as April 6-10	sorted objects according to initial consonants (block, mailbox, telephone, tractor, turtle, basket, bed, monkey, money, magnifying glass, television, feather, motorbike, donkey, bird, bear, banana, marble, ballerina, magnet, binoculars, boat, butterfly, top) --moved on to grouping objects in different sets.	worksheets involving sets of objects like	Michael mastered discriminate classification characteristics logical to all objects in a set.
		one-to-one correspondence		and	
		equivalence	used sets of objects in the room; i.e., pencils, crayons, pens, paper, marbles, etc. to identify common characteristics and	worksheets with addition problems involving one and two more than; i.e., 6 +1, 3 +2, etc.	Michael mastered discriminate classification differences not in a set of objects
		greater than or less than			Michael mastered recognition of sets.
					Michael had trouble identifying what was meant by one-to-one correspondence. By the end of the week he grasped what was meant; however, more drill and practice was reported to be necessary.

Figure 4.5 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
			differences--rolling, square, long, very small, colored, textured, etc. The objects mentioned were jointly written on paper--child wrote, but teacher helped with spelling, etc.		Michael mastered what was meant by greater than, less than, and equivalence; however, when it came to answering how many greater than he had problems.
			sets of play cups and saucers; child was asked to determine by matching whether there were the same number of cups as saucers		
			chinese checker board with six sets of ten marbles with different colors in each set; was asked to determine whether there was enough marbles for six people to play the game; was asked to determine which player would have more marbles		
			worksheets with illustrations with two sets of equivalent and non-equivalent sets of objects--child was asked to identify which set had more than and less than the other		
April 27-May 1	5	meaningful counting from one to thirty cardinality set union	used textbook pages to count numbers from one through 110. counted the number of children in the school and also number of teachers given sets of objects in boxes with marbles, poker chips, etc., having different cardinalities, was asked to identify the number of objects in each set. rapidly exposed flash cards presenting elementally grouped objects one through nine; answers called out numeral 1-9; drew appropriate number of pictures disjoint sets of physical objects from EXLP kit; asked to join the sets with common characteristics.	same as April 20-24 textbook involving the use of the numberline in demonstrating addition and subtraction sentences such as $5 + \underline{\ } = 7$ and $4 - \underline{\ } = 2$ textbook involving addition and subtraction as inverse processes such as $7 - 4 = 3$ and $3 + 4 = 7$	Michael mastered meaningful counting. Michael mastered cardinality of sets. Michael mastered set union. Michael continued having trouble with addition and subtraction facts with sums and minuends less than ten. Michael was able to demonstrate that addition and subtraction are inverse processes with the numberline but appeared to have trouble explaining why.

Figure 4.5 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Resource Assignment	Results
May 4-8	4	writing addition sentences using words and numerals addition facts with sums less than or equal to nine subtraction facts with minuends less than or equal to nine	poker chips and paper-and-pencil; first teacher manipulated and Michael wrote then teacher wrote and Michael manipulated--Michael also wrote sentences using words and sentences made flashcards of addition facts with sums less than or equal to eighteen--worked on facts less than ten made flashcards of subtraction facts with minuends less than or equal to eighteen--worked on facts with minuends less than six 100 addition facts on timed test (31/34 of 100 in five minutes)	same as April 20-24 worksheets using counting man with problems like 3 + 2 = 5 worksheets involving the zero property in addition and subtraction problems like 4 + 0 = ___ and 2 - 2 = ____ textbook exercises emphasizing addition and subtraction problems with sums and minuends less than ten; i.e., 3 + 5 = ____ and 9 - 5 = _____	Michael needed more help with writing addition sentences using words and numerals. He had difficulty spelling the words and there were indications that Michael had still not realized that numerical sentences are shorthand for talk and writing. Michael continued to use the numberline and used hash marks to compute answers to addition number facts with sums greater than four. Michael needed more encouragement with his number facts. He apparently saw the exercises as nothing but memorization because he missed anything to do with the zero property, commutative property and subtracting a number from itself. Michael became quite discouraged and on one occasion the session had to be cut short.
May 11-15	5	same as May 4-8 addition and subtraction as inverse processes	same as May 4-8 addition and subtraction fact number wheels; drill and practice on number facts. The idea is to move the dial at random, make a stab at the answer, then check the answer on the back.	same as May 4-8 worksheet using counting man emphasizing tens and ones in addition and subtraction	Michael continued having trouble with the idea that numerical sentences are shorthand sentences. When using anything but the flash cards and number wheels on basic facts Michael reverted to the use of hash marks and numberline to determine his answers. Michael continued being quite hesitant to respond during the early part of the week. However, when the number wheels were introduced he became fascinated because they had the same things on them as did his flash cards, but were more fun. He worked diligently with them each day for up to seven or eight minutes and then tired. At that time he was able to move on to the other activities. Michael did make the connection between addition and subtraction as inverse processes. Michael had real problems with the counting man in working with tens and ones. His answers were merely following a model established by the teacher with apparently little or no conceptualization. Wadded up paper indicated frustration.

Figure 4.5 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
May 18-22	2	addition facts with sums less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen	counting wheels using addition and subtraction number facts with sums and minuends less than or equal to eighteen bingo using addition and subtraction number facts with sums and minuends less than or equal to eighteen	worksheets using single pencils and bundles of ten pencils to designate tens and ones worksheets with addition and subtraction problems with numerals less than five worksheets requiring counting by twos and tens worksheets requiring understanding of one more than property in written out and numerical form	Michael continued the excitement with working the number wheels. He had most of the addition facts memorized up to 4 + 4. He still had problems with the commutative property. Michael had also memorized most of the number facts to 4 - 4. It appeared that he gained an interest in learning the number facts as a challenge or game. Michael had problems in the bingo game since Karen and Cinthia were his competitors. (He had been asked to join the game.) Michael apparently saw little or no need to think of numbers in groups of tens and ones. His work was full of errors and doodling marks. Michael seemed to comprehend the one more than property concept. However, he continued having trouble with written word sentences. He did seem to realize that number sentences are shorthand for written or spoken words
May 25-29	3	addition facts with sums less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen	counting wheels, flash cards, and bingo using addition and subtraction facts with sums and minuends less than or equal to eighteen	worksheets requiring addition of three and four numbers with one and two digits not requiring regrouping 86 addition facts with sums less than ten on timed test; 15/16 in two minutes worksheets with counting sum - filling in missing numbers like 3 + □ = 7	Michael still enjoyed working with counting wheels and at times the flash cards, but quit working on the bingo games. Perhaps it was because he could not compete with Karen and Cinthia. It was decided to separate the children again and teach Michael completely on his own again. Michaels responses were very slow. Michael was able to do the column addition problems. There were indications that he computed the problems by rote (hash marks).

Figure 4.5 (cont'd.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
June 1-5	3	addition facts with sums less than or equal to eighteen subtraction facts with minuends less than or equal to eighteen counting wheels with one, two, three and four on the addition dial associative and commutative properties of addition	56 addition facts on timed tests with sums less than ten--best two minute response for week was 34/36 45 subtraction facts on timed test with minuends less than ten--best two minute response for week was 20/23 flash cards with addition and subtraction facts with sums and minuends less than ten to build speed and accuracy poker chips used to demonstrate commutative properties of addition as they relate to addition and subtraction; first teacher manipulated, child wrote; later teacher wrote, child manipulated	worksheet of addition and subtraction facts worksheets with three numeral addition with sums less than ten worksheets with counting man demonstrating tens and ones worksheets with bundles of sticks demonstrating tens and ones worksheets requiring computation of two digit addition and subtraction problems requiring regrouping and non-regrouping	Michael needed more work on addition and subtraction facts with sums and minuends less than ten. He knew the addition facts with sums up to seven but continued having problems with sums greater than seven. Michael did not conceptualize what was meant by the commutative property of addition. He apparently did not realize the necessity of commutative and associative properties since he could not operationalize it using the basic number facts. Also, he had no idea why 4-3 was not 3 - 4 and (5 - 1) - 3 was not 5 - (1 - 3). Michael was able to compute addition and subtraction problems not requiring regrouping. He was able to compute two digit addition problems requiring regrouping but he could not figure out why $\begin{array}{r} 23 \\ -\ 7 \\ \hline \end{array}$ was not 34. Michael finally caught on to what was meant by the counting man using the left hand for tens digit and right hand for the ones digit. He could not transfer that to bundles of sticks.
June 8-12	3	same as June 1-5	same as June 1-5	same as June 1-5	same as June 1-5 - Except, Michael mastered commutative property of addition as it relates to addition. He mastered addition facts with sums less than ten. He even caught on to the idea using basic facts.
June 15-19	2	same as June 1-5	same as June 1-5	same as June 1-5	same as June 8-12 - Except, Michael caught on to what was meant by commutative property of addition as related to subtraction.
June 22-26	1	took achievement test	revised researcher-designed diagnostic-achievement test	none	See post-test data in Table 4.1

Michael demonstrated mastery of many concepts related to addition and subtraction. Whereas Michael had difficulty understanding ways of discriminating set membership at the onset of the study, he was able to do so at the conclusion. Also, he had mastered the following higher level tasks: "recognition of sets", "one-to-one correspondence", "greater than or less than", "equivalence", "meaningful counting", "empty set", "write meaningful symbols for 0-9", "cardinality", "set union", "set separation", "commutative and associative properties of union", "principle of addition", "identify element of addition", "addition facts with sums less than ten", "commutative property of addition", "principle of subtraction", "addition and subtraction as inverse processes", and "subtraction facts with minuends less than ten".

Michael was unable to respond to the attitude scale in the manner of the other children. He was unable to read the words and had difficulty following the tester's instructions. Therefore, he was asked to respond in the following manner: "I will give you two words at a time. Tell me how you feel about doing arithmetic by using one of the two words. If you feel kind of inbetween, tell me that. If you do not understand what the words mean, ask me, I will explain them to you." For each of the bi-polar adjectives in the semantic differential attitude scale he was asked the following question: "How does your ability to do arithmetic make you feel; _____ or _____ or inbetween?" His reactions to the attitude scale indicated a definite positive trend (see Scale 4.5). However, he did realize that one of his main problems was his slowness. Michael's homeroom teacher mentioned that the child had progressed beyond expectations. Even though Michael was slow he stayed with tasks until he understood what he was doing. There were definite indications that Michael became displeased with himself during some of the instructional exercises and he reverted to his "I won't try" attitude. However, Teacher A patiently worked with Michael in working with manipulative materials demonstrating the mathematical actions involved in the tasks and Michael responded positively.

Case Study of Robert

Biographical Sketch

Robert Myers was seven-and-one-half years old when this study was conducted. He was slender, small for his age, and look malnourished. Robert was susceptible to diseases and this was substantiated when he was out of school for over a month during the instructional phase of the study. Some of the adjectives used to describe Robert and his behavior were: hyperactive, impulsive, restless, demanding, hostile, anxious, disorganized, incompetent, mentally retarded, disobedient, angry, uncooperative, selfish, spoiled, immature, short attention, lying, delinquent behavior, running away from home, fire setting, bed wetting, loves alcohol especially vodka, gas sniffing, always hungry, etc. Some of the disoriented types of behaviors were somewhat alleviated when it was discovered that Robert was being sedated heavily and was taken off the prescribed drugs by the staff doctor.

Robert was the oldest of four boys. Mr. and Mrs. Myers were unable to decide how to handle his weasling, whining manner of getting his own way. Mrs. Myers, who was reported to be a housewife, was described by the staff at Edgefield Lodge as an impatient person who rewarded Robert's whims just to get him to quit whining. Mr. Myers, a foreman of a major food store in Portland, had a drinking problem. When under the influence of alcohol Mr. Myers reportedly mistreated Mrs. Myers and the boys. The parents had been warned that unless conditions in the family changed the boys would be temporarily or permanently removed from the home. This threat was carried out and Robert was placed in Edgefield Lodge.

Robert was thought to be mentally retarded; however, many of his actions suggested that he was continually being ostracized because of his intimidating, fondling, teasing, deceptive mannerisms. As a result, he had difficulty keeping friends and often used money, candy, and other items to bribe children to play with him. His reading and arithmetic skills were minimal. In a school progress report Robert was reported to be working at a kindergarten level of performance. He was able to count from one to ten and to follow dot-to-dot number sheets with the use of a numberline as a guide. He was also having difficulty attaching cardinality to sets of objects.

Diagnostic-Manipulative Arithmetic Program

Preassessment of Arithmetic Competencies. The results from the revised researcher-designed diagnostic-achievement test, reported in Table 4.1, showed that Robert had not mastered the tasks "knowledge of concrete manipulative materials and activities" and "discriminate and classify characteristics logical to all objects in a set." Robert had not mastered "discriminate and classify differences not in a set of

of objects', recognition of sets", etc. (see Appendix A).

Instructional Program. Instruction began for Robert on April 3, 1970. Teacher A was his teacher for the period of time he was involved with the special arithmetic sessions. Michael was the other child with whom Teacher A was working at the same time Robert was receiving special instruction. Robert got sick three weeks into the instructional phase of the study and was later dropped from further investigation because of his continued illness.

Figure 4.6 Instructional Program Provided for Robert
From April 3, 1970 to May 22, 1970

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignments	Results
April 3	1 .	discriminate and classify characteristics to all objects in a set discriminate and classify differences not in a set of objects	HELP items 2-5 (see Appendix C) color identification (ten colors) bead design (sequencing patterns) block design (shape patterns) bolt board (size seriation)	worked with materials in HELP kit in a non-directed manner	Robert was able to identify the names of all the primary colors (red, yellow, blue), secondary colors (orange, purple, green), brown and black. However, he did not know pink ("lighter red") and gray ("silvery"). Robert was able to duplicate bead patterns. However, on more than one occasion he reversed the patterns but later changed it. He was able to string the bead patterns by memory, again at times reversed the sequencing. Robert enjoyed creating his own designs. Robert enjoyed working with the bolt board (wooden stand, nine holes of diminishing sizes fitted with bolts and nuts to match). He was able to take apart, mix, assemble and explain the principles he used. Robert explained that he liked to work with his Dad when his Dad would let him. He often got into trouble because of taking things apart. Robert was able to arrange nine colored blocks to match a pattern of similar size to the blocks and also match miniaturized patterns of similar designs. It was difficult to get him to follow prearranged patterns; he wanted to make his own designs. Robert became tired of an activity easily. After seven or eight minutes on exercises he would not remain at-task without encouragement. However, with the bolt board he had to be encouraged to move on. Prior to testing sessions Robert had been allowed to work with the items in the HELP kit without any teacher help.
April 6-10	3	same as April 3 recognition of sets	HELP items 7, 9-11 (see Appendix C) numbers (more, less, many, some); writing a name (letter configurations, etc.); auditory perception (name of small toys, sorted, renaming sets); social interaction (identified rules, agreements in conflict situation.	same as April 3 worked on dot-to-dot puzzles using numerals 1-30 worked on writing numerals from 1-20 without references	Robert was able to place the plastic pieces of different lengths and colors in seriated order according to length. There was some question whether he attached the concept of order to the length or to the numeral written on the plastic pieces. Each plastic piece had the appropriate number of outlined squares as indicated by the accompanied numeral and

Figure 4.6 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
					length of object. Robert was able to count to ten but did not know what was "two less than six", for instance.
					Robert had reversal problems with his letters. He worked diligently to print his name. In fact, he used upper and lower cased letters. However, he got letters reverse.
					Using beginning consonant sound Robert was unable to discriminate when the sounds were very close; i.e., s - z. However, he was able to think of names of objects with similar initial consonants.
					Robert was very social conscious. He verbalized fluently the rules and standards established in his homeroom and previous school classrooms. However, when attempting to get him to behave according to the rules he verbalized that as a different matter.
					Robert was really quite verbal, though much of his vocabulary was limited. His descriptions of objects understandably took the form of his previous experiences. This week Teacher B worked with Robert using color crayons. He developed quite a vocabulary for color. Previously he differentiated colors but attached other names to them. Mixing colors was revealing to him to form other colors.
					Robert worked on bead designs and became quite proficient at sequencing the designs and making up his own designs. There was every indication of the child having limited experiences. When exposed Robert learned quickly.
					Robert thrived on the personal attention he and Michael were receiving. He commented regularly about how he enjoyed playing and drawing.
					Robert had difficulty with following sequenced numbers in dot-to-dot puzzles.

Figure 4.6 (con't.)

Week	No. of Days	Tasks	Materials and Activities	Homeroom Assignment	Results
April 13-17	0	none (spring vacation)	none	none	none
April 20-24	3	same as April 6-10	sorted objects according to initial consonants (block, mailbox telephone, tractor, turtle, basket bed, monkey, money, magnifying glass, television, feather, motor bike, donkey, bird, bear, banana, marble, ballerina, magnet, binoculars, boat, butterfly, top)--moved on to grouping objects in different sets. used sets of objects in the room, i.e., pencils crayons, pens, paper, marbles, etc. to identify common characteristics and differences - rolling, square, long, very small, colored, textured, etc. - objects mentioned were jointly written on paper; Robert wrote but teacher helped with spelling, etc.	same as April 6-10 worked on filling in 3 x 10 matrix counting on sequencing numbers--one more than idea worked on assigning cardinality to sets of objects one to ten and writing appropriate numbers	Robert became quite proficient discriminating characteristics of sets and was having some success in discriminating and classifying differences not in a set. His vocabulary was limited regarding commonly accepted word usages. However, that did not seem to bother him as he used words that were convenient to describe what he saw and handled. At times it was difficult to keep Robert on a trend of thought. For breaks he was allowed to work with the design blocks, block board, etc.
April 27-May 1	1	same as April 20-24	same as April 20-24	none	Robert became ill and was out most of the week.
May 4-8 (111)	0	none	none	none	none
May 11-15 (111)	0	none	none	none	none
May 18-22 (111)	0	none	none	none	Robert was dropped from the study at the end of this week. He had measles, mumps, and colds which kept him out of school until after the first week in June.

CONCLUSIONS

This study investigated the arithmetic achievement and attitude changes of institutionalized emotionally disturbed children of ages seven to eleven years. They were given an arithmetic program which involved diagnosis of mathematics skills and instruction based upon the diagnosed skills. The program emphasized looking for mathematical patterns, and manipulation of physical objects. Emphasis was also placed upon the children's problem-solving understanding of mathematical actions rather than associative memorization of rules.

A task analysis format was followed to develop hierarchies of skills for the operations of addition and subtraction, and each skill was behaviorally defined. A diagnostic-achievement test was constructed based upon defined behaviors and items required the children to: (1) manipulate objects illustrating number sentences, (2) write number sentences describing the actions performed on the objects by the tester, and (3) solve paper-and-pencil problems. The initial diagnostic test was modified and adapted to the needs of the children in the study. Sub-sections of the test included computational skills, knowledge of concepts, and applications of processes.

Twelve children from Edgefield Lodge Residential Center for Emotionally Disturbed Children at Troutdale, Oregon were tested with the revised researcher-designed diagnostic-achievement test. Six of those children were identified for case investigation. One of the six subjects selected to be studied was dropped from further investigation approximately halfway through the instructional period because of continued illness. The diagnostic-achievement test was used to determine the selection of subjects and to identify where instruction began for each child. Behavioral objectives served as criteria for task mastery during the instructional phase and subjects were provided with an average of thirty-eight days of twenty-to-thirty minute instructional periods. At the conclusion of the study the revised diagnostic-achievement test was given again to measure arithmetic achievement. Also, subjects were given a semantic differential attitude scale to determine attitude change, along with data gathered from the children's teachers. Two of the five certified teachers at Edgefield Lodge were selected to participate in the study and several training sessions were provided to clarify what the study was about.

Case Study Summary

Bart. At the beginning of this study Bart lacked mastery of many tasks leading to a problem solving understanding of the operations of addition and subtraction. He had mastered all of the tasks identified

in Appendix A for addition and subtraction by the conclusion of the study. Also, Bart had mastered simple multiplication facts and had an understanding that multiplication is repeated addition.

At the beginning of the study Bart expressed a negative attitude toward arithmetic, which was summarized in a statement recorded in his cumulative folder made by him, "I hate arithmetic." However, as Bart became more successful in solving mathematics problems using his newly acquired skills his attitude improved somewhat. Although his responses to the bi-polar items in Scale 4.1 were not completely positive, there were indications from what he said and did that he felt better about his ability to do arithmetic at the conclusion of the study than he did at the beginning.

David. When this study was initiated David had not mastered many of the tasks beyond an understanding of the principles of addition and subtraction. He had difficulty with the properties associated with adding and subtracting and knew few of the basic number facts. However, as the study concluded, David had mastered all of the tasks leading to a problem solving understanding of addition, except he continued having difficulty reading story problems. Regarding subtraction, David still had difficulty with the higher level basic facts, with borrowing, and with sifting through word problems to differentiate essential from non-essential information.

David's attitude toward his ability to do arithmetic had improved from very negative to somewhat positive, as indicated in Scale 4.2. Further, at the conclusion of the study, he was willing to try new things without the teacher's help whereas he would not attempt them prior to the study. David was able to use many of the manipulative ideas utilized during the instructional phase of the study in his homeroom assignments to prove his work. His teacher mentioned that prior to the study David had difficulty remaining at-task and did his work only to get it out of the way. At the conclusion of the study he was not satisfied until a task was performed properly.

Karen. At the onset of the study, Karen demonstrated mastery of the tasks leading to an understanding of the principles of addition and subtraction, except for an understanding of set union and ability to write out the word names of numerals. She was not quite sure why the associative property of union did not change the final outcome when sets were joined. She was confused and became frustrated when presented with higher level tasks. At the conclusion of the study Karen had difficulty with writing word names of numerals, higher level addition facts, and regrouping (carrying) when computing, but had mastered the other tasks for addition mentioned in the hierarchy in Appendix A. She continued having difficulty with comparable tasks in subtraction; i.e., higher level subtraction facts, regrouping (borrowing), etc.

Apparently Karen did not see the necessity for regrouping (borrowing)

when computing subtraction problems and had difficulty setting up and solving subtraction word problems because of her inability to discriminate essential from non-essential items.

Karen's attitude toward her ability to do arithmetic improved considerably from the beginning to the end of the study. Karen indicated semi-positive reactions to the attitude scale (scale 4.3) at the conclusion of the study, which were different from her completely negative responses toward anything to do with mathematics at the beginning of the study. Since mathematics was Karen's most difficult subject in school, she had adopted a defensive, negative, avoidance reaction toward it. This changed as the instructional phase progressed to where she would attempt new and different tasks without the continued assistance and/or approval of the teacher. On several occasions, Karen's homeroom teacher mentioned that Karen was so proud of her accomplishments that she explained her mathematical discoveries to others, whereas, she had not done this previously.

Cinthia. When the study was initiated Cinthia demonstrated mastery of the prerequisite tasks to the principles of addition and subtraction of whole numbers, except for commutative property of union. By the conclusion of the study she had mastered all addition tasks except for setting up and solving word problems. Cinthia was still having difficulty with the more complex number facts and higher order tasks in subtraction when the study was completed.

Cinthia expressed herself positively about her ability to do mathematics at the beginning of the study. However, it was soon confirmed that she verbalized better than she actually performed. As the study progressed Cinthia's attitude fluctuated from positive to negative. Cinthia's response on the attitude scale was somewhat more positive on the last day of instruction; however, there were indications that she still fostered negative responses about her ability to do arithmetic. In contrast, to indicate Cinthia's feeling toward the special sessions, she made it a point to greet the researcher each time he entered the school with some comment about a new discovery she had made about mathematics.

Michael. Michael made substantial achievement gains in arithmetic during the study. As reported in Table 4.1 Michael had not mastered tasks beyond discriminating and classifying characteristics of sets of objects. When the study ended Michael had mastered readiness tasks and all tasks leading up to addition and subtraction facts with sums and minuends less than ten, except for writing word names for the symbols 0-9. Also, Michael had mastered regrouping ones to tens and the commutative property of addition.

Michael was unable to respond to the attitude test in Scale 4.5 without the assistance of the tester. Nevertheless, he answered the bi-polar items in the manner prescribed in Appendix C. His attitude toward his

ability to do arithmetic at the conclusion of the study had improved considerably from the beginning of the study.

Discussion

It is difficult to classify the emotional behaviors of the six children studied into concise categories. For instance, Quay, Morse, and Cutler (1969) characterized emotionally disturbed children into the following categories: unsocialized, unattached, and inadequacy-immaturity. The descriptions of the children's behaviors in this study illustrated the over-lapping of the emotional behaviors characteristic to those categories. Perhaps if more were known about the effects of emotional disturbances on learning arithmetic tasks, more information could be provided regarding the lasting effects of a program such as this study.

Children who completed the study demonstrated substantial gains in their arithmetic achievement and attitudes toward their ability to do arithmetic from the beginning to the end of the study. These conclusions were drawn from the results reported in Table 4.1 and the descriptions of attitude changes in the individual case studies. Whereas each child demonstrated lack of mastery of many prerequisite skills leading to a problem solving understanding of more difficult addition and subtraction problems at the beginning of the study, many of those more complex tasks were mastered by the children at the conclusion of the study. Also, children who began the instructional phase of the study on tasks listed toward the bottom of the hierarchies in Appendix A were not able to catch up with children who began with tasks further up the hierarchies; in fact, each child was encouraged to learn at his own rate, with the possible exception of Karen who was taught (on numerous occasions) using the same lesson format as the one for Cinthia.

Most of the time the children worked on tasks which were difficult but not impossible for them to master. When the diagnostic feature of this study was followed carefully, continued success in progressively more difficult tasks was evidenced. When the diagnostic feature was not followed there were indications of frustration in the children's behavior. Furthermore, attitude changes corresponded with how individual children expressed their ability to be successful doing the arithmetic tasks on which they worked, rather than mastery of all the tasks in the hierarchies.

Piaget (1969) and Bruner (1964) emphasized the role of maturation in learning. There were indications that the children in this study exhibited immature behaviors and corresponding low arithmetic achievement patterns. In conjunction with the hypothesis that many emotionally disturbed children of comparable ages and similar behavior and learning problems are

slow maturers, the investigator found that the children studied were retarded in their abilities to classify, conserve, and reverse concrete operations. For example, Robert, Michael, and Karen were unable to conserve number at the beginning of the study, whereas that ability is usually attained by children who are six years old.

From the case data available to this investigator, there were indications that teachers at Edgefield Lodge traditionally utilized available commercial materials which emphasized the presentation of paper-and-pencil, drill-and-practice exercises in a non-hierarchically arranged sequence of activities. Arithmetic instruction based on foundational, readiness types of active learning experiences rather than continued paper-and-pencil drill-and-practice exercises seems more appropriate for them.

Teachers who participated in the study followed the instructional paradigm proposed by Popham and Baker (1970) which is identified in Table 1.1. The four steps of the instructional pattern were: specification of objectives, pre-assessment, instruction, and evaluation. However, at times one or more of the steps were not followed to completion. For instance, at times Teacher A did not fully understand what was meant by the behavioral objectives in Appendix B and did not even refer to them to determine levels of task mastery possessed by Karen, Cinthia, Robert, or Michael prior to moving on to higher level tasks. Also, at times, Teacher A had difficulty doing the following: sequencing instructional tasks, providing a variety of manipulative activities and materials designed for specific tasks, utilizing continued diagnosis with the specific behavioral criteria in Appendix B as points of task mastery, using paper-and-pencil for drill-and-practice exercises sparingly, emphasizing the children's problem solving understanding over rote memorization of rules, and collecting case data by the instructional day using the prescribed forms in Appendix D. However, Teacher B was more regular in following the above mentioned procedures. Also, both teachers helped children look for mathematical patterns and utilized games and active involvement exercises during the instructional phase of the study.

Conclusions of the Study

The following conclusions were drived from the study:

1. Children who completed the study expressed more positive attitudes toward their abilities to do arithmetic at the conclusion of the study than at the beginning.

2. Children who completed the study mastered arithmetic tasks at a more rapid rate than they had previously.

3. Children who were presented tasks in an hierarchical manner learned those tasks more rapidly than children who were presented tasks in a somewhat random order.

4. While tasks in the hierarchies could be performed at a rote or associative level, a background of using the tasks in a wide variety of manipulative situations helped to provide for principle or problem-solving levels of task mastery.

5. Because of the small number of subjects and the number of uncontrollable variables, the results of this mathematical program are tenuous, and possibly affected by one or more of the following conditions: (a) maturation rates; (b) emotional disturbances and effects on learning; (c) changes in instruction; (d) effect of teacher training; (e) effects of attitude changes; (g) diagnosis of specific skills; and (h) interaction with peers during arithmetic instruction.

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APPENDIX A

Mathematics Hierarchies

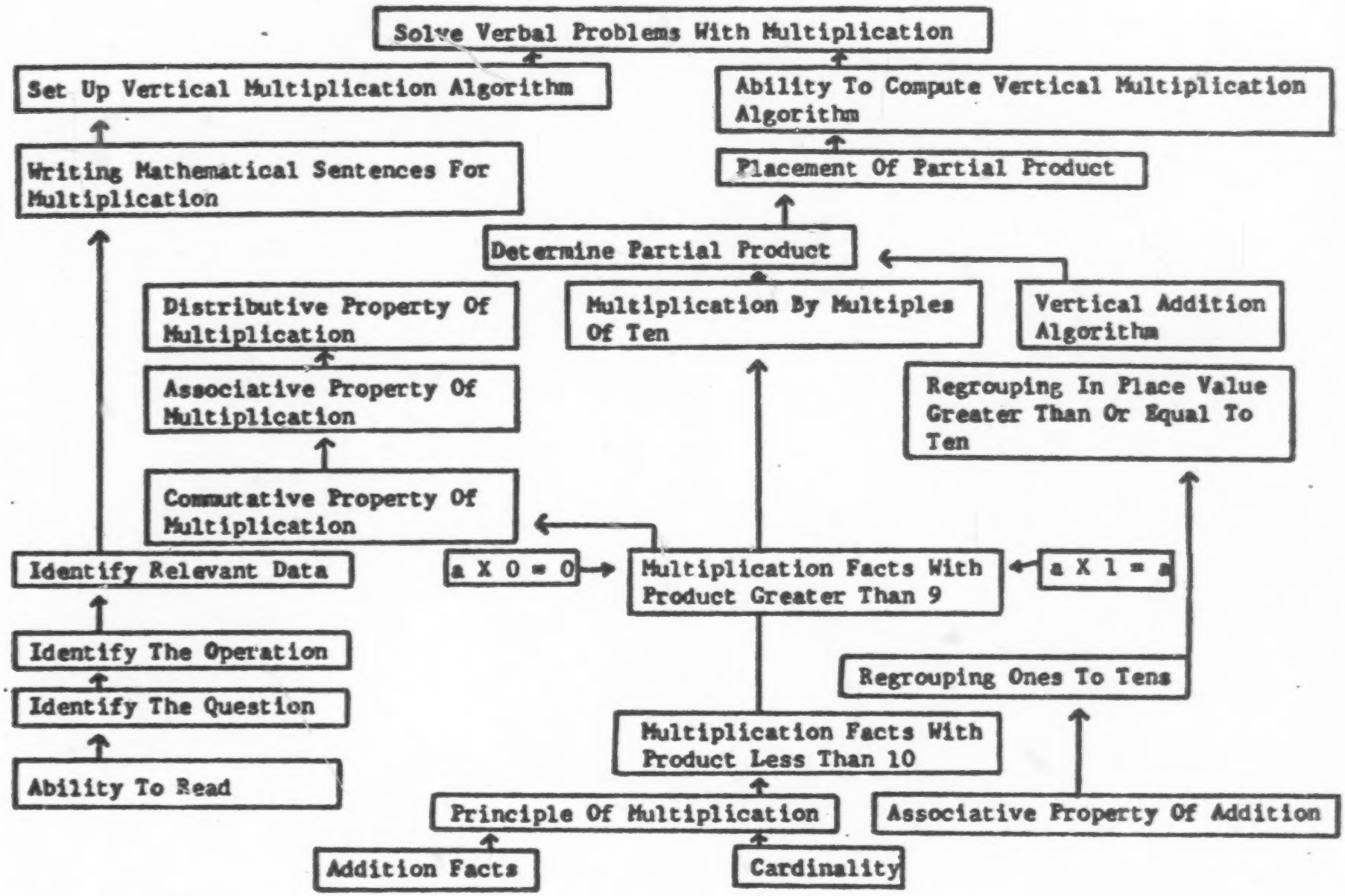
Hierarchy A_1: Addition of Whole Numbers

Hierarchy A_2: Subtraction of Whole Numbers

Hierarchy A_3: Multiplication of Whole Numbers

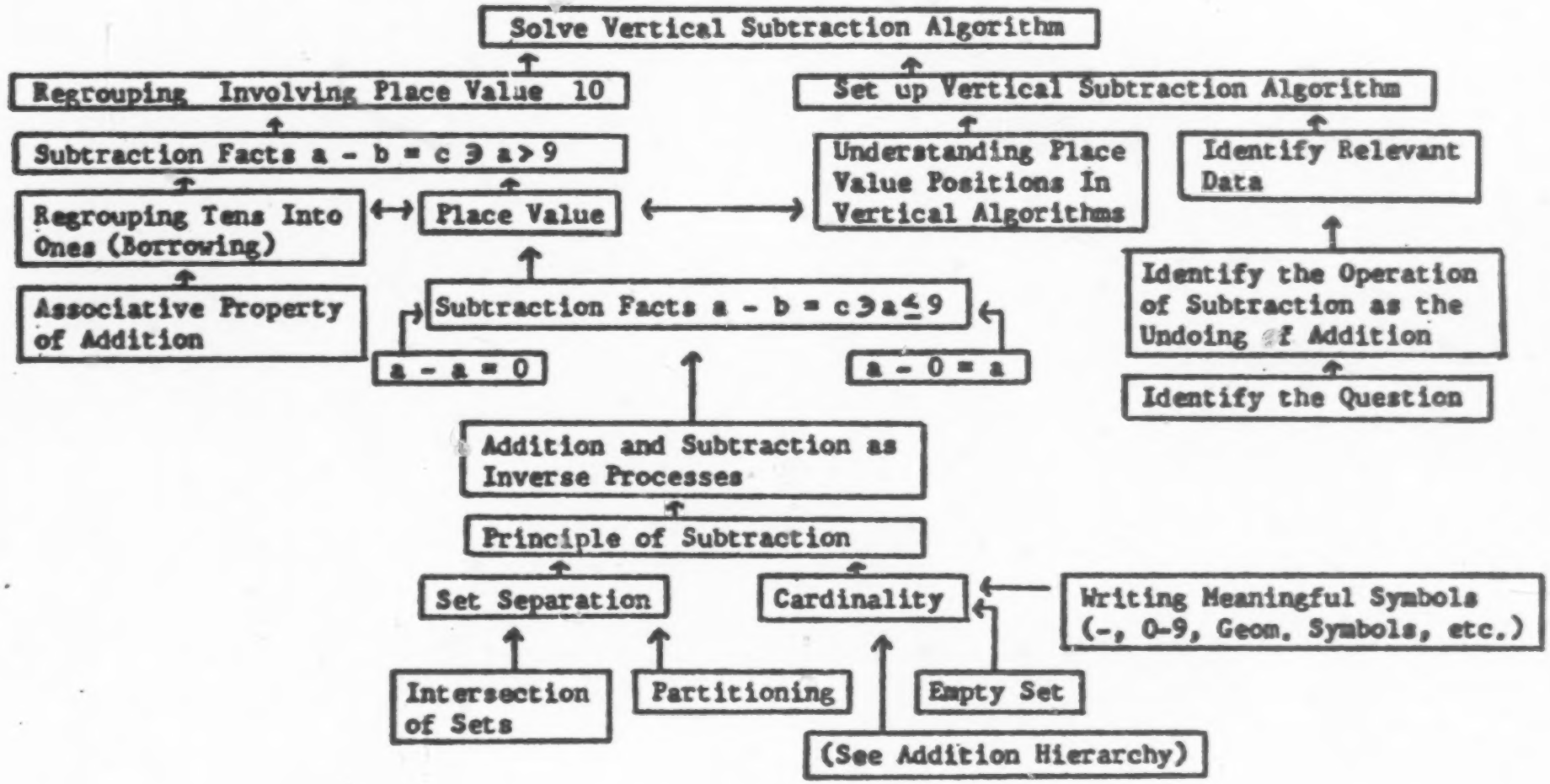
APPENDIX A_3

MULTIPLICATION OF WHOLE NUMBERS: BASIC HIERARCHY


Solve Verbal Problems With Multiplication
Set Up Vertical Multiplication Algorithm
Ability To Compute Vertical Multiplication Algorithm
Writing Mathematical Sentences For Multiplication
Placement Of Partial Product
Determine Partial Product
Distributive Property Of Multiplication
Multiplication By Multiples Of Ten
Vertical Addition Algorithm
Associative Property Of Multiplication
Regrouping In Place Value Greater Than Or Equal To Ten
Commutative Property Of Multiplication
Identify Relevant Data
a X 0 = 0
Multiplication Facts With Product Greater Than 9
a X 1 = a
Identify The Operation
Regrouping Ones To Tens
Identify The Question
Multiplication Facts With Product Less Than 10
Ability To Read
Principle Of Multiplication
Associative Property Of Addition
Addition Facts
Cardinality

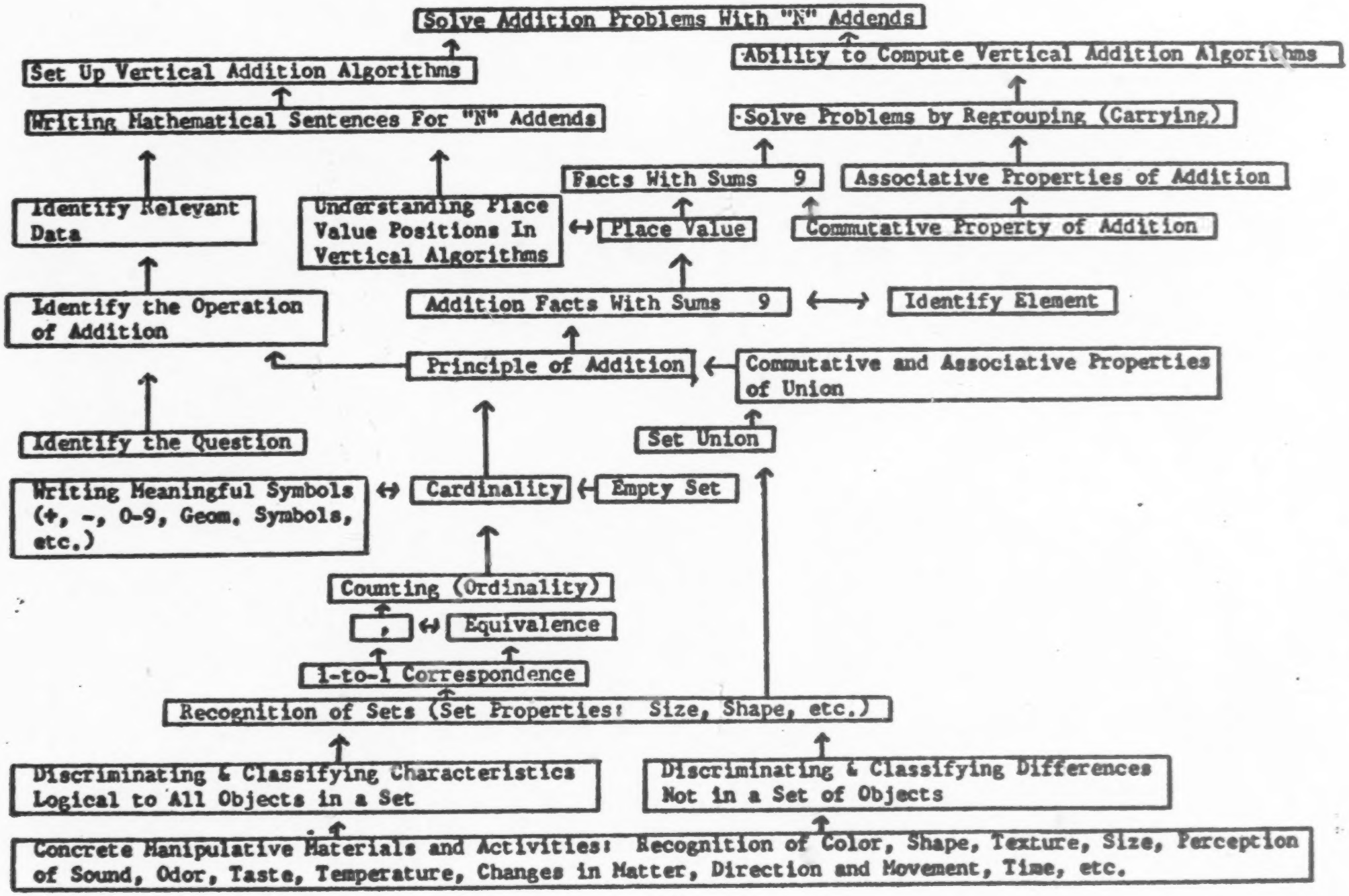
APPENDIX A_2

SUBTRACTION OF WHOLE NUMBERS: HIERARCHY


Solve Vertical Subtraction Algorithm
Regrouping Involving Place Value 10
Set up Vertical Subtraction Algorithm
Subtraction Facts a - b = c ∋ a > 9
Understanding Place Value Positions In Vertical Algorithms
Identify Relevant Data
Regrouping Tens Into Ones (Borrowing)
Place Value
Identify the Operation of Subtraction as the Undoing of Addition
Associative Property of Addition
Subtraction Facts a - b = c ∋ a ≤ 9
Identify the Question
a - a = 0
a - 0 = a
Addition and Subtraction as Inverse Processes
Principle of Subtraction
Set Separation
Cardinality
Writing Meaningful Symbols (-, 0-9, Geom. Symbols, etc.)
Intersection of Sets
Partitioning
Empty Set
(See Addition Hierarchy)

APPENDIX A_1

ADDITION OF WHOLE NUMBERS: BASIC HIERARCHY


Solve Addition Problems With "N" Addends
Set Up Vertical Addition Algorithms
Ability to Compute Vertical Addition Algorithms
Writing Mathematical Sentences For "N" Addends
Solve Problems by Regrouping (Carrying)
Facts With Sums 9
Associative Properties of Addition
Identify Relevant Data
Understanding Place Value Positions In Vertical Algorithms
Place Value
Commutative Property of Addition
Identify the Operation of Addition
Addition Facts With Sums 9
Identify Element
Principle of Addition
Commutative and Associative Properties of Union
Identify the Question
Set Union
Writing Meaningful Symbols (+, -, 0-9, Geom. Symbols, etc.)
Cardinality
Empty Set
Counting (Ordinality)
,
Equivalence
1-to-1 Correspondence
Recognition of Sets (Set Properties: Size, Shape, etc.)
Discriminating & Classifying Characteristics Logical to All Objects in a Set
Discriminating & Classifying Differences Not in a Set of Objects
Concrete Manipulative Materials and Activities: Recognition of Color, Shape, Texture, Size, Perception of Sound, Odor, Taste, Temperature, Changes in Matter, Direction and Movement, Time, etc.

APPENDIX B

Behavioral Objectives

B_1: Addition of Whole Numbers

B_2: Subtraction of Whole Numbers

B_3: Multiplication of Whole Numbers

APPENDIX B_1: Behavioral Objectives for Addition of Whole Numbers

Algorithm Model:

(Addend)	4
+(Addend)	+5
(Sum)	9

Solve Addition Problem With "N" Addends

Given five written word (i.e., sentence in story problem form) and five non-written (i.e., orally presented and/or physically manipulated) problems involving "N" addends, the learner must do the following: (1) recognize those problems which involve additive situations; (2) correctly construct a number sentence for each addition problem; and (3) compute each correct answer with 100% accuracy.

Set Up Vertical Addition Algorithm

Given ten addition number sentences involving sets of "N" addends with "N" digits, the learner must correctly align each addend into a vertical algorithm with 100% accuracy.

Write Number Sentence For Addition Problem

Given five written word and five non-written addition problems involving "N" addends, the learner must correctly construct a number sentence for each problem with 100% accuracy.

Identify Relevant Data

Given five written word and five non-written addition problems involving "N" addends, the learner must identify the addends needed to correctly answer each problem with 100% accuracy.

Identify Operation of Addition

Given five written and five non-written addition problems, the learner must specify that the operation of addition is necessary to identify the answers for each additive situation with 100% accuracy.

Identify Questions

Given five written word and five non-written addition problems involving "N" addends, the learner must answer the following question for each problem with 100% accuracy: "What is the question this problem is asking you to answer?"

Ability to Read Problem

(It is assumed by the researcher that each child at this level in the hierarchy must be able to read the printed words. If a child cannot perhaps it is necessary for the teacher or someone to read the problem for him.)

Compute Vertical Addition Algorithm

Given fifteen vertical addition algorithms requiring both regrouping and non-regrouping and involving "N" addends that vary in number of digits, the learner must compute the algorithms with at least 90% accuracy.

Solve Addition Problems by Regrouping

Given ten vertical addition algorithms involving "N" addends that require regrouping (i.e., carrying ones to tens, tens to hundreds, etc.), the learner must correctly compute each sum and explain what he has done with 100% accuracy.

Addition Facts With Sums Greater Than Nine and Less Than or Equal to Eighteen

Given rapid flash card presentations of forty-five addition facts whose sums are greater than nine but less than or equal to eighteen, the learner must correctly call out each sum with 100% accuracy.

A Given a worksheet with forth-five vertically written facts whose sums are greater than nine but less than or equal to eighteen, the learner must correctly write each sum in two minutes or less with 100% accuracy.

Associative Property of Addition

Given five written addition problems involving three or more addends, the learner must show that $(a + b) + C = a + (b + C)$ by grouping three or more sets of physical objects to form two sets whose sums are equal to those represented by the numerals slipped into the above formula with 100% accuracy.

Given five situations with three or more sets of physical objects, when shown how the objects may be grouped in more than one way such that $(a + b) + c = a + (b + c)$ and shown that the sum is not affected, the learner must correctly describe the grouping action and write one or more appropriate number sentences that represent the grouping action with 100% accuracy.

Commutative Property of Addition

Given ten written and five non-written addition problems involving

two addends and given a set of physical objects, the learner must show that a + b = b + a with 100% accuracy.

Understand Place Value

Given five sets of physical objects whose cardinality is greater than ten, the learner must align the objects into groups of ones, tens, etc. and write a numeral representing the composite number of each set with 100% accuracy.

Given ten written numerals with varying place values represented, the learner must identify the name of and distinguish between the place values occupied by digits in the numerals with 100% accuracy.

Given five written numerals and five orally presented numbers, the learner must identify what is meant by each numeral or number by showing representations on the abacus, the pocket chart, or both with 100% accuracy.

Addition Facts With Sums Less Than or Equal to Nine

Given rapid flash card presentations of forty-five addition facts whose sums are less than or equal to nine, the learner must correctly write each sum in two minutes or less with 100% accuracy.

Given a wroksheet with forty-five vertically written addition facts whose sun is less than or equal to nine, the learner must correctly write each sum in two minutes or less with 100% accuracy.

Identity Element of Addition

Given five written addition problems involving untaught numerals which are greater than ten, having zero as one of the addends, the learner must name the sum with 100% accuracy.

Given five number sentences in the form a + 0 = ☐ such that "a" is less than ten and given a set of physical objects, the learner must correctly align the appropriate number of objects to form a set to represent the sum with 100% accuracy.

Principle of Addition

Given a series of five physical manipulations involving the joining of two or more disjoint sets of objects, the learner must correctly describe the action represented by writing an appropriate number sentence for each with 100% accuracy.

Given five number sentences, each involving two or more addends, the learner must correctly manipulate physical objects to demonstrate the action called for by each addition sentence with 100% accuracy.

Associative Property of Union

Given five manipulative problems involving the union of more than two disjoint sets of physical objects, such that (a + b) + c = a + (b + c), the learner must respond in each case to the question, "Does switching the order of the objects around change the total number of objects and why?"

Commutative Property of Union

Given five manipulative problems involving the union of two disjoint sets of physical objects, such that a + b = b + a, the learner must correctly respond in each case to the question, "Does switching the order of the objects around change the total number of objects and why?"

Set Union

Given ten physical objects arranged into two or more disjoint sets, and told to join the objects to form one set, the learner must physically join each of the disjoint sets into a single set and identify the members of the new set by their characteristics with 100% accuracy.

Given sketches of five disjoint sets of objects and given the instructions to join the sets, the learner must draw sketches of the joined set in each of five problems with 100% accuracy.

Cardinality

Given rapidly exposed flash card presentations of elementally grouped objects, one through nine (exposure less than one second), the learner must correctly call out the appropriate number represented by each card with 100% accuracy.

Given exposures to nine ungrouped sets of physical and/or sketched objects, the learner must correctly write the numeral to represent the number of each set with 100% accuracy.

Write Meaningful Symbols for 0-9

Given orally presented and/or physical objects representations of the number 0-9 in random order, the learner must correctly write the appropriate numeral and word for each number with 100% accuracy.

Empty Set

Given five orally presented problems requiring the counting of set members of empty sets, the learner must correctly indicate that there are no objects in each set by calling out the number "zero" when asked "How many _____ are there in the set of _____?" with 100% accuracy.

Meaningful Counting

Given five sets of objects of varying number, the learner must point to each object in turn while calling the appropriate number in sequence with 100% accuracy.

Given a set of discs with numerals 1-30 written on them, and given a set of thirty additional physical objects, the learner must match one-to-one and in an ordinal sequence the set of discs with the other set, while calling the appropriate number with 100% accuracy.

Given a set of thirty sketched objects on a worksheet, the learner must number the objects using an ordinal sequence of one through thirty with 100% accuracy.

Greater Than Or Less Than

Given five written and five manipulative problem situations involving two or more sets of physical objects, the learner must identify those sets which are not equivalent to a given set and correctly identify whether they are greater than or less than the given set through one-to-one correspondence with 100% accuracy.

Equivalence

Given five manipulative problems involving two or more sets of various physical objects, including two or more sets of objects in each problem with the same cardinality, the learner must identify which two sets of each problem have the same number of objects in them by one-to-one correspondence with 100% accuracy.

Given five written problem situations, each involving the representations of two or more sets of physical objects, the learner must identify which sets of objects have the same number by using one-to-one correspondence with 100% accuracy.

One-to-One Correspondence

Given five situations involving two sets of physical objects, the learner must match each object from one set with a corresponding object from the other set with 100% accuracy.

Given five worksheets with two sets of items shown on each, the learner must match the items in one set with the items in the other set by drawing lines between single items of each set with 100% accuracy.

Recognition of Sets

Given five sets of physical objects and five sets of sketched objects, the learner must orally list the characteristics to each set with 100% accuracy.

Given defined characteristics for each of five sets and given several sets of physical objects and several sets of sketched objects, the learner must identify each defined set by specific characteristics with 100% accuracy.

Discriminate and Classify Characteristics Logical to All Objects in a Set

Given five scrambled sets of physical objects, with each set having one or more defined characteristics, the learner must discriminate all objects that have the given characteristics.

Discriminate and Classify Differences Not in a Set of Objects

Given five scrambled sets of physical objects, with each set having one or more defined characteristics, the learner must discriminate all objects that do not have the given characteristics.

Knowledge of Concrete Manipulative Materials and Activities

Given physical materials, the learner must describe the following characteristics as they occur: color, shape, size, texture, temperature, design, odor, taste, etc.

Note: In the testing situation each characteristic must be specifically identified.

APPENDIX B_2: Behavioral Objectives for Subtraction of Whole Numbers

Algorithm Model:

(Minuend)	9
-(Subtrahend)	-4
(Difference)	5

Solve Vertical Subtraction Algorithm

Given six written words (i.e., sentence in story problem form) and six non-written (i.e., orally presented and/or physically manipulated) problems involving numerals with "N" digits, a given minuend, a given subtrahend, and the unknown difference, such that each given minuend is equal to the sum of the subtrahend and the difference, the learner must do the following: (1) recognize those problems which involve subtractive situations; (2) correctly construct a given number sentence for each subtraction problem; and (3) compute the correct answer to each of the problems with 100% accuracy.

Set up Vertical Subtraction Algorithm

Given ten subtraction number sentences involving numerals with "N" digits (e.g., 4265 - 172 = ; 27 - 18 = ; 87 - 9 = ; etc.), each number sentence having a minuend, a subtrahend, and a difference, the learner must correctly align each numeral into an appropriate vertical algorithm form with 100% accuracy.

Write Number Sentence for Subtraction Problem

Given six written word and six orally presented subtraction problems involving a minuend, a subtrahend, and a difference, the learner must correctly construct a mathematical sentence (e.g., 175 - 69 = , etc.) for each problem with 100% accuracy.

Identify Relevant Data

Given six written word and six non-written subtraction problems, the learner must identify the given numbers needed to solve each problem with 100% accuracy.

Identify Operation of Subtraction

Given six written words and six non-written subtraction problems (using the three forms of subtraction situations such that 9 - 4 = s, 9 - s = 5, and s + 4 = 9), the learner must specify that the operation of subtraction is necessary to identify the answers for each subtractive situation with 100% accuracy.

Identify Question

Given six written word and six non-written subtraction problems, the learner must answer the following question for each problem with 100% accuracy: "What is the question this problem is asking you to answer?"

Ability to Read Problem

(It is assumed by the researcher that each child at this level in the hierarchy must be able to read the printed words. If the child cannot, perhaps it is necessary for the teacher or someone to read the problem for him.)

Compute Vertical Subtraction Algorithm by Regrouping Place Value Greater Than or Equal to Ten

Given ten vertical subtraction algorithms requiring regrouping (i.e., carrying hundreds to tens, thousands to hundreds, etc.) and non-regrouping of numerals that vary in number of digits, the learner must compute the algorithms and explain what he has done with 100% accuracy.

Subtraction of a Number from Itself

Given five written subtraction problems having the same number for minuends and subtrahends, the learner must name the difference with 100% accuracy.

Given five number sentences, such as a - a = ____, and given a set of physical objects, the learner must correctly group the appropriate number of objects to form a set to represent the difference with 100% accuracy.

Subtracting Zero

Given five written subtraction problems involving untaught numerals, having zero as the subtrahend, the learner must name the difference with 100% accuracy.

Given five number sentences, such as a - 0 = ____, and given a set of physical objects, the learner must correctly group the appropriate number of objects to form a set to represent the difference with 100% accuracy.

Subtraction Facts, Such that "a" Subtract "b" Equals "c" and "a" Is Greater Than Nine and Less Than or Equal to Eighteen

Given rapid flash card presentations of the eighty-one subtraction facts whose minuend is greater than nine but less than or equal to

eighteen and whose subtrahend is less than or equal to nine (e.g., 18 - 7 = 15; etc.), the learner must correctly call out each difference with 100% accuracy.

Given a worksheet with forty-five vertically written facts whose minuends are greater than nine but less than or equal to eighteen and whose subtrahend is less than or equal to nine, the learner must correctly write each difference in two minutes or less with 100% accuracy.

Compute Vertical Subtraction Algorithm by Regrouping Tens into Ones

Given ten vertical subtraction algorithms requiring regrouping of tens into ones in order that the difference can be identified, the learner must compute the algorithms and explain what he has done with 100% accuracy.

Commutative and Associative Properties of Addition

(see: Addition of Whole Numbers)

Understand Place Value

(see: Addition of Whole Numbers)

Subtraction Facts, Such That "a" Subtract "b" Equals "c" and "a" Is Less Than or Equal to Nine

Given rapid flash card presentations of fifty-five subtraction facts whose minuend is less than or equal to nine and whose subtrahend is less than or equal to the minuend (e.g., 9 - 7 = ☐; 7 ; etc.), the learner must correctly call out each difference with 100% accuracy.

Given a worksheet with forty-five vertical written facts whose minuends are less than or equal to nine and whose subtrahends are less than or equal to the minuends, the learner must correctly write each sum in two minutes or less with 100% accuracy.

Addition and Subtraction as Inverse Processes

Given ten vertical addition problems and given the instructions to show that subtraction is the "undoing" of addition, the learner must write the appropriate vertical subtraction algorithm that corresponds to the undoing of each given vertical addition problem with 100% accuracy.

Principle of Subtraction

Given five manipulative situations involving the separation of one disjoint set from a larger set of physical objects (e.g., three from

seven, etc.), the learner must correctly describe the action called for by writing the appropriate number sentence for each situation with 100% accuracy.

Given five number sentences, each involving a subtractive situation, the learner must correctly manipulate physical objects to demonstrate the action called for by each subtraction sentence with 100% accuracy.

Set Separation

Given ten physical objects forming one set that is made up of two or more disjoint sets, when told to separate the disjoint sets from the larger set, the learner must physically separate the sets and identify the members of each specified disjoint set by the given characteristics with 100% accuracy.

Given sketches of five disjoint sets of objects forming one large set and given the instructions to separate the sets, the learner must draw sketches of the five disjoint sets with 100% accuracy.

Note: From Set Separation on (i.e., Cardinality, etc.), the addition hierarchy and the subtraction hierarchy are identical.

APPENDIX B_3: Behavioral Objectives Multiplication

Solve Verbal Problems With Multiplication

Given a set of fifteen one- and two-step written problems involving at least two factors, the learner must be able to do the following:

(1) recognize those problems which involve multiplicative situations
(2) correctly construct a mathematical sentence for each multiplicative problem
(3) set up a vertical multiplication algorithm
(4) compute the correct answer to each of the problems with 90% of the problems computed correctly.

Set up Vertical Multiplication Algorithm

Given ten multiplication number sentences involving sets of two factors with "N" digits, the learner must correctly align each factor into a vertical algorithm with 100% accuracy.

Write Mathematical Sentences

Given five written word and five orally presented multiplication problems involving "N" factors, the learner must correctly construct a multiplication equation for each problem with 100% accuracy.

Identify Relevant Data

Given five written word and five non-verbal multiplication problems involving "N" factors, the learner must identify the factors needed to correctly answer each problem with 100% accuracy.

Identify the Operation

Given five written word and five non-verbal multiplication problems, the learner must specify that operation of multiplication is necessary to identify the answers for each multiplicative situation with 100% accuracy.

Identify the Questions

Given five written word and five non-verbal multicplication problems involving "N" factors, the learner must answer the following question for each problem with 100% accuracy: "What is the question this problem is asking me to answer?"

Ability to Read

(It is assumed by the researcher that each child at this point in the hierarchy must be able to read the printed words.)

Computing Vertical Multiplication Algorithm

Given fifteen vertical multiplication algorithms requiring both regrouping and non-regrouping and whose factors vary in number of digits, the learner must compute the algorithms with 95% accuracy.

Placement of Partial Product

Given ten vertical multiplication algorithms for which the partial products have been computed, the learner must correctly align the written partial products for each algorithm and explain what he has done with 100% accuracy.

Determine Partial Product

Given ten verticla multiplication algorithms requiring both regrouping and non-regrouping and involving two factors that vary in number of digits, the learner must correctly compute all of the partial products in each problem and explain what he has done with 100% accuracy.

Computing Vertical Addition Algorithm

Given fifteen vertical addition algorithms requiring both regrouping and non-regrouping and involving "N" addends that vary in number of digits, the learner must compute the algorithms with 95% accuracy.

Multiplication by Powers of Ten

Given five written numerals and/or five orally presented numbers to be multiplied by powers of ten, the learner must correctly compute each product and explain what he has done with 100% accuracy.

Given five written untaught numerals and five orally presented untaught numbers to be multiplied by powers of ten, the learner must correctly compute each product by suffixing an appropriate number of zeros to each given numeral with 100% accuracy.

Distributive Property of Multiplication over Addition

Given sets of five written and five physically manipulated multiplication problems involving two factors, one factor expressed as a sum of two addends, the learner must correctly write equations and manipulate objects with 100% accuracy to show that the product of the factors is equal to the sum of the products obtained from the factor times each addend.

Regrouping in Place Value Greater Than or Equal to Ten

Given five written numerals and five orally presented numbers, the learner must correctly write an equation using the associative property for addition to form a different written representation of each numeral with 100% accuracy.

Given five written numerals and five orally presented numbers, the learner must correctly show two different representations on the abacus for each numeral with 100% accuracy.

Associative Property of Multiplication

Given five situations involving sets of three factors, the learner must correctly write equations for each set of factors with 100% accuracy showing two different groupings of the factors and a product which is the same for each grouping.

Given sets of five written multiplication problems involving three factors whose product is less than 100, the learner must show that (a x b) x c = ax (bxc) by physically manipulating objects into two equivalent sets with 100% accuracy.

Commutative Property of Multiplication

Given sets of ten written and five physically manipulated multiplication problems involving two factors, the learner must show that a x b = b x a with 100% accuracy.

Multiplication by One

Given five written multiplication problems involving untaught numerals, having one as a factor, the learner must name the proudct with 100% accuracy.

Given the number sentence, a x 1, and a given set of physical objects, the learner must correctly group the appropriate number of objects to form a set to represent the product with 100% accuracy.

Multiplication by Zero

Given five written multiplication problems involving untaught numerals, having zero as one of the factors, the learner must name the product with 100% accuracy.

Given the number sentence, a x 0, and a given set of physical objects, the learner must correctly group the appropriate number of objects to form a set to represent the product with 100% accuracy.

Multiplication Facts Whose Products Are Greater Than Nine

Given rapid flash card presentations of the fifty-eight multiplication facts whose products are greater than nine, the learner shall correctly call out each product with 100% accuracy.

Given the fifty-eight written multiplication facts, the learner must correctly write each product in two minutes or less with 100% accuracy.

Regrouping Ones to Tens

Given five sets of objects whose cardinality is greater than ten, the learner must align the objects into groups of ones, tens, etc., and write a numeral representing the composite number with 100% accuracy.

Given five situations in which objects are grouped into ones, tens, etc., the learner must regroup the objects into one less group of tens and ten more ones and correctly describe the action represented by writing appropriate number sentences for each with 100% accuracy.

Given five written numerals and for five orally presented numbers, the learner must correctly write an equation showing the use of associative property of addition to form a different written representation of each numeral with 100% accuracy.

Associative Property of Addition

Given five situations with three or more sets of objects and shown how they may be grouped in more than one way without affecting the sum, the learner must correctly describe the grouping action by writing a number sentence with 100% accuracy.

Given sets of five written addition problems involving three or more addends, the learner must show that $(a + b) + c = a + (b + c)$ by grouping three or more sets of objects to form two sets whose sum is equal to two sets grouped in another fashion with 100% accuracy.

Multiplication Facts Whose Product Is Less Than Ten

Given rapid flash card presentations of the forty-two multiplication facts whose product is less than ten, the learner must correctly call out each product with 100% accuracy.

Given the forty-two written multiplication facts, the learner must correctly write each product in two minutes or less with 100% accuracy.

Principle of Multiplication

Given a series of five manipulations involving the joining of equivalent sets, the learner must correctly describe the action represented

by writing appropriate number sentences for each with 100% accuracy.

Given five number sentences involving two factors each, the learner must correctly manipulate objects to demonstrate the action called for by each sentence with 100% accuracy.

Addition Facts

Given rapid flash card presentations of the 100 addition facts, the learner must correctly call out each sum with 100% accuracy.

Given the 100 written addition facts, the learner must correctly write each sum in two minutes or less with 100% accuracy.

APPENDIX C

C_1: Revised Researcher-Designed Diagnostic-Achievement Test

C_2: Attitudinal Scale (Semantic Differential)

C_3: KELP (Kindergarten Evaluation of Learning Potential)

APPENDIX C_1: Instructions for Administering Revised Researcher-Designed Diagnostic-Achievement Test

READ THE COMMENTARY AND INSTRUCTIONS TO THE FOLLOWING DIAGNOSTIC-ACHIEVEMENT TEST BEFORE PROCEEDING!

"If I do not try I will not fail" is an attitude emotionally disturbed children often have toward academic learning situations, including confrontations with arithmetic. This diagnostic test is designed for emotionally disturbed children who are having difficulty understanding mathematical abstractions related to addition and subtraction of whole numbers. The issue is not where such attitudes and difficulties began, but how we can help alleviate them and help such children grow experientially from them. It is the responsibility of the tester to encourage successful mathematical experiences, as well as to determine the functional level of mathematics competencies that the learner being tested possesses. However, it must be remembered that this instrument is not a teaching device, buth rather a testing instrument to diagnose specific arithmetic deficiencies prior to instruction and/or measure arithmetic achievement following instruction.

Arithmetic tests often consist of paper-and-pencil exercises and seldom involve the manipulation of physical objects. There is often a gap between a child's ability to write down abstract symbols in arithmetic sentences or algorithmic notation and his ability to visualize the mathematical actions using concrete forms. Therefore, this test is concerned with not only whether the learner can write down arithmetic abstractions correctly and compute correct answers, but also whether he knows what he must do to work similar problems in a variety of problem solving situations.

The test is organized so that the samples of test items relate to the hierarchy of skills identified in Appendix A and defined in Appendix B. The test is written using an hierarchical arrangement of subordinate, concomitant, and higher-level tasks. The items on the test require the children to: (1) mainpulate objects illustrating number sentences, (2) write number sentences describing the actions performed on the objects, and (3) solve paper-and-pencil exercises. Also, sub-sections of the test include computational skills, knowledge of concepts, and applications of processes. The test is designed to allow the tester as much autonomy as possible and still maintain the hierarchical structure of skills to be tested. Also, the test is designed to allow the child being tested the opportunity to respond to questions and concepts in more than one way.

All of the test items need not be presented to each child. It is important to make an approximate placement on the test, therefore, if the child being tested is unfamiliar to the tester, begin by asking him to count to 25, or some other appropriate task. If the child is familiar

to the tester then begin on the diagnostic test at least two items below the level the tester is sure the child will be successful. Depending on the rapidity and accuracy of the child's responses to the tester's initiating question, place the child somewhere along the continuum of test items. As one can readily determine, it is necessary for the tester to be familiar with all test items. Also, choose a beginning task that involves a composite of the immediate prerequisite tasks.

If the child being tested can perform the initial task with mastery, then the tester should assume that he can probably perform most, if not all prerequisite tasks. If the tester has any reason to doubt mastery of prerequisite tasks it is suggested that the tester probe to determine if the child is actually able to perform them. Suggestions are given in the context of the test, however, this does not preclude other probings initiated by the tester, as long as the interchange does not become a teaching session. The tester should move along the continuum of test items and try every fifth item. Follow this procedure until a frustrating and non-mastered task is identified. Return to the last task successfully performed and test sequential test items until the tester identifies the level at which the child needs to begin instructional activities, i.e., the level at which the child is not able to demonstrate a mastery of three consecutive tasks. Keep in mind, that if the child has mastered cardinality and subordinate tasks, then both addition and subtraction test items need to be tested.

All questions initiated by the tester and all child responses should be written down in as much detail as possible. This means that if there is not enough available space allowed for comments on the test the tester should write additional responses on the back of the paper, with the number of the item that is commented on circled for future reference. In most cases, children respond to the questions in a manner consistent with their logical understandings and/or misconceptions. Perhaps instructional procedures can be followed to reinforce correct understandings.

Keeping in mind that attention span of emotionally disturbed children is often a problem, perhaps this test should be presented at more than one sitting. The testing activities and procedures have been designed to maintain the attention of the children by providing variety of activities and materials. However, if the child being tested becomes too restless, choose another testing period in the very near future to continue the testing.

APPENDIX C_1: Revised Researcher-Designed Diagnostic-Achievement Test

Teacher's Guide

Discriminate Concrete Manipulative Materials and Activities

1. Given: Teacher physically places a set of physical objects (i.e., attribute blocks with different shapes, sizes, and colors) in front of the learner.

   Directions: "What different (colors,shapes, sizes) do you see in this set of blocks?" (The appropriate answer should be that which the tester is attempting to get the learner to discriminate. The tester should carefully probe and record both questions and responses.)

Comments: _____ COLOR☐

_____ SHAPE☐

_____ SIZE☐

_____

_____

Discriminate and Classify Characteristics Logical to All Objects in a Set

2. Given: Teacher physically places a set of physical objects with different shapes and sizes but with the same color (yellow).

   Directions: "What is it about these blocks that are the same?"

Comments: _____ ☐

_____

3. Given: Teacher physically places a set of physical objects, with different sizes and colors but with the same shape. (diamond, etc.)

   Directions: "What is it about these blocks that are the same?"

Comments: _____ ☐

_____

4. Given: Teacher physically places a set of physical objects, with different colors and shapes but with the same size. (small)

Directions: "What is it about these blocks that are the same?"

Comments: _____ ☐

_____

Discriminate and Classify Differences Not in a Set of Objects

5. Given: Teacher physically places a set of physical objects, with different shapes, sizes, and colors (i.e., attribute blocks) in front on the learner.

Directions: "Put the blocks that are not characteristic in my hand." (red, etc.)

Comments: _____ ☐

_____

_____

Recognition of Sets

6. Given: Teacher physically places three disjoint sets of objects with differing characteristics in front of the learner.

Directions: "Show me the set that has this many (hold up three, four, and then zero fingers) sides." If you have questions why the learner chose the sets the way he did ask, "Why did you choose this set (point)?" (Be sure to record the child's response.)

Comments: _____ ☐

_____

_____

One-to-One Correspondence

7. Given: Two disjoint sets drawn on the attached worksheet.

Directions: "Show how these sets are the same number by drawing lines between the boys and the balls."

Comments: _____ ☐

_____

_____

8. Given: Two disjoint sets drawn on the attached worksheet.

   Directions: "Show how these sets are not the same number by drawing lines between the girls and the boys."

Comments: _____ ☐

_____

_____

Equivalence

9. Given: Two disjoint sets of ten objects.

   Directions: "Do these sets have the same number of objects in them?" (Are these sets equivalent?) "Show how you found out?"

Comments: _____ ☐

_____

_____

10. Given: Two disjoint sets of objects; nine objects and eleven objects drawn on the attached worksheet.

    Directions: "Do these sets have the same number of objects in them?" (Are these sets equivalent?) "Show how you found out?"

Comments: _____ ☐

_____

_____

Greater Than or Less Than

11. Given: Four disjoint sets of objects; eleven objects, seven objects, seven objects, and five objects.

    Directions: "Which set has more objects than the other sets? Which set has less objects than the other sets?"

Comments: _____ ☐

_____

_____

Meaningful Counting

12. Given: Teacher physically places one set of fifteen objects and one set of thirty objects in front of the learner.

    Directions: "Count out loud the number of objects in this set." (Point to the sets as you want the child to count them.)

Comments: _____ /7

_____

_____

13. Given: Sets of objects away from the testing table.

    Directions: "Go to the cupboard and count out 11 objects and bring them to the testing table." (If he fails to respond correctly, ask for 7.)

Comments: _____ /7

_____

_____

Empty Set

14. Given: For the following questions the learner must give an appropriate response:

    Directions: "Tell me the names of people in the room who have green hair." "Draw a picture of the set of live elephants in the room." "Make up a set that has the same number as the problems I have just given you."

Comments: _____ /7

_____

_____

Write Meaningful Symbols for 0 - 9

15. Given: Teacher identifies appropriate space on the worksheet and holds up (eight, five, and then zero) fingers.

    Directions: "Write the word for the number of fingers I am showing you." "Write the numeral for the same number of fingers."

Comments: _____ ☐

_____

_____

Cardinality

16. Given: Teacher physically places arrays of various sets of 15 objects in front of the learner (ungrouped).

    Directions: "How many objects are there altogether?" (Describe how the learner got his answer.)

Comments: _____ ☐

_____

_____

17. Given: Set of elementally grouped objects (dots) having cardinalities one through nine on separate flash cards (exposure less than one second and randomly presented) -- see accompanying envelope.

    Directions: "Call out the number of dots you see on each card as quickly as you can."

Comments: _____ ☐

_____

_____

18. Given: Teacher writes the numerals 7 and 4 on the worksheet (one numeral at a time).

    Directions: "Draw a picture of this many objects (point to the written numeral that is intended)."

Comments: _____ ☐

_____

_____

Set Union

19. Given: Two disjoint sets of physical objects having cardinalities of 4 and 5.

Directions: "Join the objects in the three sets together by drawing them as one set." "What have you done to make them one set?"

Comments: _____ ☐

_____

_____

20. Given: Three disjoint sets of sketched and configurations on the worksheet having cardinalities of 3, 2, and 3.

Directions: "Join the objects in the three sets together by drawing them as one set." "What have you done to make them one set?"

Comments: _____ ☐

_____

_____

Set Separation

21. Given: Teacher physically places fifteen objects in front of the learner.

Directions: "Separate the fifteen objects into a set of nine objects and a set of six objects."

Comments: _____ ☐

_____

_____

Commutative Property Union

22. Given: Physical objects in arrangements such that 4 U 3 = 3 U 4.

Directions: "Is this side (point) of the equation equivalent to this other side (point)?" "Why?" (If in question try 2U5 = 5U2.)

Comments: _____ ☐

_____

_____

23. Given: Two problems of sketched objects written on the worksheet, such that 3 U 6 = 6 U 3 and 5 U 4 = 4 U 5.

Directions: "Is this side (point) of the equation equivalent to this other side (point)?" "Why?" (Each problem should be done individually.)

Comments: _____ ☐

_____

_____

Associative Property of Union

24. Given: Two problems of sketched objects written on the worksheet, such that (2U6)U4 = 2U(6U4) and 7U(2U2) = (7U2)U2.

Directions: "Is this side (point) of the equation equivalent to this other side (point)?" "Why?" (Each problem should be done individually.)

Comments: _____ ☐

_____

_____

Operation of Addition

25. Given: Teacher physically performs the joining action; 9 + 6 = 15.

Directions: "I am going to use objects to show a number sentence. Write a number sentence, like 3 + 5 = 8, to show what I have done."

Comments: _____ ☐

_____

_____

26. Given: 25 objects for the child to use to demonstrate the following directions.

Directions: "Using these objects, show how to find the answer to the number sentences 3 + 4 = ☐ and 7 + 1 + 5 = ☐ ."

Comments: _____ ☐

_____

_____

Identity Element

27. Given: 25 objects for the child to use to demonstrate the following directions.

    Directions: "Using these objects, show how to find the answer to the number sentence, 8 + 0 = ☐ ." (If in doubt try 0 + 5 = ☐ .)

Comments: _____ ☐

_____

_____

28. Given: Written problems on the worksheet.

    Directions: "Write the answer to each problem." (86 + 0 = ☐ , ☐ = 0 + 28, and 9223 + ☐)

Comments: _____ ☐

_____

_____

Addition Facts Whose Sum is Less than Ten

29. Given: 30 flash cards with addition facts satisfying the specification identified above.

    Directions: "Call out the answer to each flash card as quickly as you can." (Record the problems the child misses.)

Comments: _____ ☐

_____

_____

30. Given: Worksheet with 30 addition facts satisfying the specification identified above.

    Directions: "Work as quickly as you can. You may work for one minute."

Comments: _____ ☐

_____

Regrouping Ones and Tens

31. Given: Teacher places twenty-four ungrouped objects in front of the learner.

    Directions: "Put the objects into groups of ten. How many ones are left over? Write a numeral showing the number of tens and the number of ones."

Comments: _____ □

_____

_____

32. Given: Problems on the worksheet.

    Directions: "Write a number sentence showing the numeral rewritten." 2 tens _ 14 ones = □ , 12 ones + 4 tens = □ .

Comments: _____ □

_____

_____

33. Given: Thirty-two objects into three sets of ten and two ones.

    Directions: "Regroup the objects so that there is one less group of tens as shown by the numeral 32. Write a number sentence to show what you did." (32 = 20 + 12 or 2 tens + 12 ones.)

Comments: _____ □

_____

_____

Addition Facts Whose Sum is Greater than Nine but Less than Eighteen

34. Given: 30 flash cards with addition facts satisfying the specification identified above.

    Directions: "Call out the answer to each flash card as quickly as you can." (Record the problems the child missed and make mention about the time it takes the child to respond with correct answers.)

Comments: _____ ☐

_____

_____

Commutative Property of Addition

35. Given: 25 objects for the child.

Directions: "Use the objects to show me that the number sentence is correct." (Write 4 + 3 = 3 + 4 on a piece of paper.)

Comments: _____ ☐

_____

_____

36. Given: Specified problems written on the worksheet (3 + 2 = 5 + 1, 2 + 3, 4 + 7; 7 + 5 = 5 + 5, 7 + 2, 5 + 7; 10 + 8 = 8 + 10, 9 + 9, 8 + 11.)

Directions: "Answer the questions by making a circle around the correct answer."

Comments: _____ ☐

_____

_____

Associative Property of Addition

37. Given: Sets of 4, 5 and 2 objects. Teacher manipulates the sets (4 + 5) + 2 and 4 + (5 + 2) to illustrate the associative property of addition.

Directions: "Write a number sentence to show what I have done."

Comments: _____ ☐

_____

_____

38. Given: 25 objects for the child.

Directions: "Use the objects to show that the following number sentence is correct: (2 + 3) + 4 = 2 + (3 + 4).

Comments: _____ ☐

_____

_____

Place Value and Regrouping

39. Given: The specified numerals and place value position to be asked about (written on the worksheet).

Directions: "What is the place value occupied by ____?" (Point to and mention 2, 4, and 3 in 243 and 0, 4, and 3, in 304.)

Comments: _____ ☐

_____

_____

40. Given: Problems on a worksheet.

Directions: "Write a number sentence to show how numerals may be borrowed to form more ones, tens, etc." (56 with a regrouping of tens to ones; 475 with a regrouping of hundreds to tens; 3829 with a regrouping from thousands to hundreds -- examples: 56 = 40 + 16; 475 = 300 + 170 + 5; 3829 = 2000 + 1800 + 20 + 9)

Comments: _____ ☐

_____

_____

41. Given: Problems on the worksheet.

Directions: "Find the answer to each problem below." (70 + 18 = ☐ ; 400 + 5 + 180 = ☐ ; 4000 + 2600 + 40 + 9 = ☐)

Comments: _____ ☐

_____

_____

Compute by Regrouping and Non-regrouping

42. Given: Worksheet containing the problems.

Directions: "Find the answer for each problem."

(No regrouping required)

35	21	231
+24	6	12
	132	102
	+ 40	+ 4

(Regrouping required)

96	106	9173
+47	1372	1202
	5	+4568
	809	
	+ 14	

Comments: _____

_____

_____

Setting up and Solving Word Problems Involving Addition

43. Given: Simple word problems involving addition of whole numbers.

Directions: "Find the answers to the following story problems." Danny and his three friends dropped marbles on Mrs. Roberson's floor. Danny dropped 15 marbles, Jim dropped 6 marbles, Billy dropped 10 marbles, and Tom dropped 22 marbles. How many marbles did the boys drop altogether?

A. "Please read the problem out loud." (If the learner cannot read the problem, the teacher should read it for him.)

Comments: _____

_____

B. "What question is this problem asking you to answer?"

Comments: _____

_____

C. "How would you find the answer to this problem?" "Would you use division, multiplication, addition, or subtraction to find the answer?"

Comments: _____

_____

D. "Point out what numbers you would use to find your answer."

Comments: _____

_____

E. "Write a number sentence like 2 + 3 + 1 = □, that tells me that you know how to find the answer to this problem." (Write the example down for the child to see. When the child is finished writing his number sentence ask him to read it to you.)

Comments: _____ □

_____

F. "Write a number sentence like $\begin{array}{r} 2 \\ 5 \\ +4 \\ \hline \end{array}$ that tells me that you know how to find the answer to this problem."

Comments: _____ □

_____

_____

G. "What is the answer to this problem?" "Tell me how you found it."

Comments: _____ □

_____

_____

Principle of Subtraction

44. Given: Teacher physically performs the separating action; 12 take away 5.

Directions: "Write a number sentence to show what I have done."

Comments: _____ □

_____

_____

45. Given: Give the child 25 objects in two groups of ten and five ones.

Directions: "Using these objects, show how to find the answer to the following number sentence." (□ = 13 - 5; 8)

Comments: _____ □

_____

_____

Addition and Subtraction as Inverse Processes

46. Given: 20 objects for the child to demonstrate the following directions.

Directions: "Use the objects to show that addition is the opposite of subtraction." (i.e., use the written equations 6 + 8 = 14 and 14 - 6 = 8.)

Comments: _____ ☐

_____

_____

47. Given: Worksheet with problem.

Directions: "use the numerals 8, 4, and 12 to write and solve one addition number sentence." "Now use these same numbers to write one subtraction number sentence."

Comments: _____ ☐

_____

_____

Subtraction Facts with Minuend Less than Ten

48. Given: 30 flash cards with subtraction facts satisfying the specification identified above.

Directions: "Call out the answer to each flash card as quickly as you can." (Record the problems the child missed.)

Comments: _____ ☐

_____

_____

49. Given: Worksheet with 30 subtraction facts satisfying the specifications identified above.

Directions: "Work as quickly as you can. You may work for one minute."

Comments: _____ ☐

_____

_____

Commutative and Associative Properties of Addition

50. Given: The two subtraction sentences that follow.

    Directions: "Answer the following questions: Is 5 - 3 = 3 - 5?" "Why?" "Is (4 - 3) - 1 = 4 - (3 - 1)?" (The child may use physical objects to show this problem.) "Why?"

Comments: _____ ☐

_____

_____

Subtraction Facts with Minuend Greater than Nine (i.e., 17 - 9 = 8)

51. Given: 30 flash cards with subtraction facts satisfying the specification identified above.

    Directions: "Call out the answer to each flash card as quickly as you can." (Record the problems the child missed and make mention about the time it takes the child to respond with correct answers, etc.)

Comments: _____ ☐

_____

_____

Subtracting Zero

52. Given: 25 objects for the child.

    Directions: "Using the objects, show how to find the answer to each number sentence." (9 - 0 = ☐; 12 - 0 = ☐; etc.)

Comments: _____ ☐

_____

53. Given: Problems on the worksheet.

    Directions: "Write the answer to each problem." (96 - 0 = ☐; ☐ = 1289 - 0; and 4569 - ☐ = 4569.)

Comments: _____ ☐

_____

Subtracting a Number from Itself

54. Given: 25 objects for the child.

Directions: "Using the objects, show how to find the answer to each number sentence." (5 - 5 = ☐; ☐ = 13 - 13.)

Comments: _____ ☐

_____

55. Given: Three written subtraction problems, such that a - a = 0.

Directions: "Write the answer to each problem." (79 - 79 = ☐; ☐ = 3016 - 3016; 939 - ☐ = 0.)

Comments: _____ ☐

_____

_____

_____

Compute by Regrouping Tens to Ones

56. Given: Teacher grouped 22 objects into two groups of ten and two ones.

Directions: "For this problem, regroup these objects into one less group of ten objects. Then, write down, using numbers, what you have done."

Comments: _____ ☐

_____

57. Given: Five subtraction problems that require regrouping of tens to ones.

Directions: "Answer all of the following problems correctly."

$\begin{array}{r} 16 \\ -9 \end{array}$ $\begin{array}{r} 251 \\ -6 \end{array}$ $\begin{array}{r} 92 \\ -79 \end{array}$ 155 - 17 = ☐ 74 - 26 = ☐

Comments: _____ ☐

_____

_____

58. Given: Teacher grouped 24 objects into 2 groups of ten and 4 ones.

    Directions: "Show me what you would do to subtract 6 from the 24 objects by regrouping the tens and the ones."

Comments: _____ ☐

_____

Computing by Regrouping (Borrowing) and Non-regrouping

59. Given: Six subtraction problems that require regrouping of numerals with place values of ten or greater.

    Directions: "Answer all of the following problems correctly."

$$\begin{array}{r} 872 \\ -91 \end{array} \qquad \begin{array}{r} 1442 \\ -73 \end{array} \qquad \begin{array}{r} 2222 \\ -383 \end{array} \qquad 401 - 308 = \square \qquad 723 - 156 = \square \qquad \begin{array}{r} 43581 \\ -4709 \end{array}$$

Comments: _____ ☐

_____

_____

Setting up and Solving Word Problems Involving Subtraction

60. Given: Simple word problems involving subtraction of whole numbers.

    Directions: "Find the answers to the following story problem." Paul wants to make a rope for his boy scout pack. He was told that the rope cannot be longer than 25 feet. Paul decided to cut a 33 foot rope so that it is 24 feet long. How many feet of rope did Paul have to cut off the 33 foot rope?"

    A. "Please read the problem out loud." (If the learner cannot read the problem, the teacher should read it for him.)

Comments: _____ ☐

_____

    B. "What question is this problem asking you to answer?"

Comments: _____ ☐

_____

C. "How would you find the answer to this problem?" "Would you use division, multiplication, addition, or subtraction to find the answer?"

Comments: _____ ☐

_____

D. "Point out what numbers you would use to find your answer."

Comments: _____ ☐

_____

E. "Write a number sentence like 43 - 17 = ☐, that tells me that you know how to find the answer to this problem." (Write the example down for the child to see. When the child is finished writing his number sentence, ask him to read it to you.)

Comments: _____ ☐

_____

_____

F. "Write a number sentence like $\begin{array}{r} 52 \\ -31 \\ \hline \square \end{array}$ that tells me that you know how to find the answer to this problem."

Comments: _____ ☐

_____

_____

G. "What is the answer to this problem?" "Tell me how you found your answer."

Comments: _____ ☐

_____

Principle of Multiplication

61. Given: Teacher physically performs the joining action in each situation.

Directions: "Write an addition sentence to show what I have done. Now, write a multiplication number sentence."

Join 2 equivalent sets with 3 objects in each set.

(3 + 3 = 6 2 x 3 = 6)

Join 2 equivalent sets with 3 objects in each set.

(2 + 2 + 2 + 2 + 2 = 10 5 x 2 = 10)

Comments: _____ ☐

_____

62. Given: 25 objects for the child.

Directions: "Using these objects, show how to find the answer to each number sentence."

3 x 2 = ☐ (6) ☐ = 4 x 5 (20)

Comments: _____ ☐

_____

Multiplication Facts

63. Given: Worksheet with 25 multiplication facts involving a factor less than or equal to five.

Directions: "Work as quickly as you can. You may work for one minute."

Comments: _____ ☐

_____

64. Given: Worksheet with 25 multiplication facts involving a factor less than or equal to five.

Directions: "Work as quickly as you can. You may work for one minute."

Comments: _____ ☐

_____

Multiplication by Zero

65. Given: 25 objects for the child.

Directions: "Using these objects, show how to find the answer to the number sentence."

3 x 0 = ☐

Comments: _____ ☐

_____

66. Given: Worksheet with problems.

Directions: "Write the answer to each problem."

98 x 0 = ☐ ☐ = 0 x 7869 1289 x ☐ = 0

Comments: _____ ☐

_____

Multiplication by One

67. Given: 25 objects for the child.

Directions: "Using these objects, show how to find the answer to the number sentence."

5 x 1 = ☐ 12 x ☐ = 12

Comments: _____ ☐

_____

68. Given: Worksheet with problems.

Directions: "Write the answer to each problem."

97 x 1 = ☐ (97) ☐ = 7891 x ☐ = 7891 (1)

Comments: _____ ☐

_____

Commutative Property of Multiplication

69. Given: 25 objects for the child.

Directions: "Use the objects to show that the number sentence is correct."

2 x 3 = 3 x 2

Comments: _____ ☐

_____

70. Given: Worksheet with problems.

Directions: "Answer the questions by making a circle around the correct answer."

5 x 4 = 19 4 x 5 7 x 3

3 x 4 = 4 x 3 13 3 x 5

8 x 2 = 3 x 8 15 2 x 6

Comments: _____ ☐

_____

Associative Property of Multiplication

71. Given: Set of three factors on a worksheet.

Directions: "Multiply the three factors by grouping them in two different ways."

4, 5, 2 (4x5)x2 = 4x(5x2) = (4x2)x5

Comments: _____ ☐

_____

Distributive Property of Multiplication

72. Given: Worksheet of problems.

Directions: "Finish each number sentence to find the answer."

5 x 7 = 5 x (4 + 3) = ☐ + ☐ = ☐

6 x 9 = 6 x (5 + 4) = ☐ + ☐ = ☐

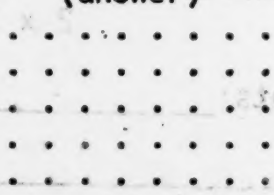
Comments: _____ ☐

_____

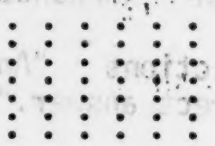
73. Given: Specified problems on a worksheet and an array.

Directions: "Show the number sentence by drawing lines on the array."

5 x 8 = 5 x (5 + 3) (answer)



7 x 6 = (4 + 3) x 6 (answer)



Comments: _____ ☐

_____

Multiplication by Powers of Ten

74. Given: Problems on a worksheet.

Directions: "Write the answer to each problem."

89 x 10 = ☐ 100 x 7896 = ☐ 1289 x 1000 = ☐

Comments: _____ ☐

_____

Determine and Place the Partial Product

75. Given: Problem on a worksheet.

Directions: "Find the part of the answer asked for in each problem below."

56 x24	(56 x 4)	1234 x319	(1234 x 300)	542 x136	(542 x 30)

Comments: _____ ☐

_____

Vertical Multiplication Algorithm

76. Given: Problems on a worksheet.

Directions: "Find the answer for each problem."

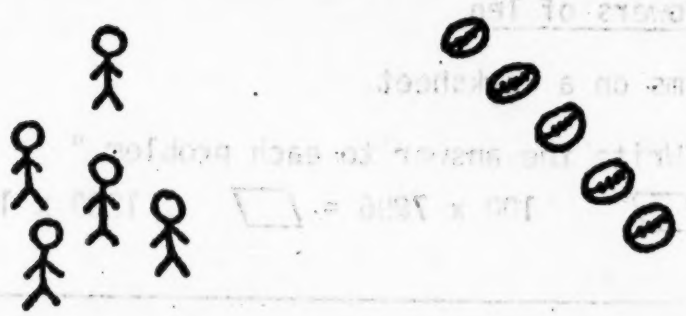
235 x 4	142 x 35	405 x 213

Comments: _____ ☐

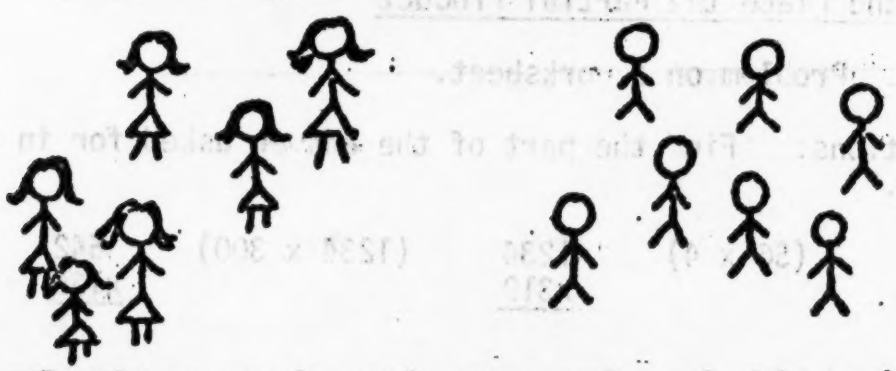
_____

Student's Worksheet: (Teachers' Copy) Name_____
Date_____

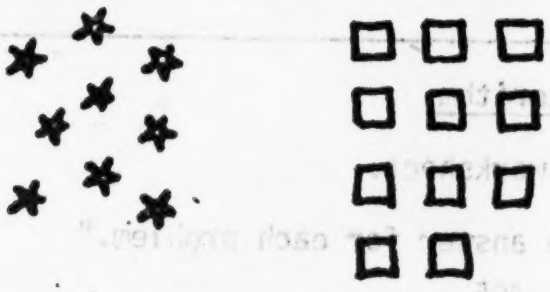
7. Show how these sets are the same number by drawing lines between the boys and the balls.



8. Show how these sets are not the same number by drawing lines between the girls and the boys.



10. Do these sets have the same number of objects in them?



YES ☐ NO ☐

14.

15. Write the word for the number of fingers the teacher shows you. Beside the word write the numeral that stands for the word.

A.

B.

C.

18. Draw a picture of 7 objects. Draw another picture of 4 objects.

20. Join the objects in the three sets together by drawing them as one set.

YOUR PICTURE

Set 1

Set 2

Set 3

23.

24.

25. Example: 3 + 5 = 8

26. Show how to find 3 + 4 = ☐ and 7 + 1 + 5 = ☐

27. Show how to find 8 + 0 = ☐ (0 + 5 = ☐)

28. Write the answer to each problem.

86 + 0 = ☐ ☐ = 0 + 28 9233 + ☐ = 9223

30. 1 minute

0	8	2	5	6	8	7	3	0	0	4	4	2	2	5
+2	+1	+6	+3	+1	+0	+2	+5	+0	+3	+3	+2	+3	+6	+1
4	1	3	3	2	3	1	5	5	2	1	8	2	1	0
+4	+7	+2	+4	+7	+3	+1	+2	+4	+2	+8	+0	+5	+2	+9

31. Write a numeral showing the number of tens and the number of ones.

32. Write a number sentence showing the correct numeral in the blank.

2 tens + 14 ones = ☐ 12 ones + 4 tens = ☐

_____ _____

33. Number sentence:

_____

35. 4 + 3 = 3 + 4

36. Answer the questions by making a circle around the correct answer.

A.	3 + 2 = 5 + 1	2 + 3	4 + 7
B.	7 + 5 = 5 + 5	7 + 2	5 + 7
C.	10 + 8 = 8 + 10	9 + 9	8 + 11

37. Write a number sentence to show what the teacher did.

_____

38. Show that (2 + 3) + 4 = 2 + (3 + 4)

39. What is the place value of: The 5 in 75691_____
4 in 96742_____
7 in 81763_____

40. Write a number sentence showing "borrowing".
A. 56 with a regrouping of tens to ones_____
B. 475 with a regrouping of hundreds to tens_____
C. 3829 with a regrouping from 1000's to 100's_____

41. Find the answer to each problem:

A. 70 + 18 = _____

B. 400 + 5 + 180 = _____

C. 4000 + 2600 + 40 + 9 = _____

42. Find the answer to each problem:

A.
$$\begin{array}{r} 35 \\ +24 \\ \hline \end{array}$$

B. 21 + 6 + 123 + 40 = _____

C.
$$\begin{array}{r} 231 \\ 12 \\ 102 \\ +\ 4 \\ \hline \end{array}$$

D.
$$\begin{array}{r} 96 \\ +47 \\ \hline \end{array}$$

E.
$$\begin{array}{r} 106 \\ 1372 \\ 5 \\ 809 \\ +\ 14 \\ \hline \end{array}$$

F. 9173 + 1408 + 4569 = _____

43. Problem: Danny and his three friends dropped marbles on Mrs. Robertson's floor. Danny dropped 15 marbles, Jim dropped 6 marbles, Billy dropped 10 marbles, and Tom dropped 22 marbles. How many marbles did the boys drop altogether?

44. Write a number sentence to show what the teacher does.

45. Show how to find the following number sentence: $\square$ = 13 - 8. You must use groups of 10 and 1's.

46. Use the objects to show that addition is the opposite of subtraction. (Suggestion: Use equations 8 + 7 = 15 and 15 - 8 or 7 = 7 or 8)

47. Use only the numerals 7, 9, and 16 to write an addition number sentence.

Use the same three numerals (7, 9, and 16) to write one subtraction number sentence.

_____

49. 1 minute

2	3	5	9	6	8	4	7	9	6
-2	-1	-3	-4	-0	-5	-2	-4	-3	-4
7	4	7	5	8	9	7	7	9	9
-3	-3	-5	-2	-7	-2	-2	-7	-7	-1
8	6	4	6	8	6	8	3	5	9
-4	-5	-1	-1	-2	-3	-8	-0	-4	-8

50. Answer the following:

A. Is 5 - 3 - 3 - 5? YES ☐ NO ☐ Why?

B. Is (4-3) - 1 = 4 - (3-1)? YES ☐ NO ☐ Why?

52. Show how to find the answer to each number sentence below by using the objects.

9 - 0 = ☐ 12 - 0 = ☐

53. Write the answer to each problem.

A. 96 - 0 = ☐

B. ☐ = 1289 - 0

C. 4569 - ☐ = 4569

54. Show how to find the answer to each number sentence below by using the objects.

5 - 5 = ☐ ☐ = 13 - 13

55. Write the answer to each problem.

A. 79 - 79 = ☐

B. ☐ = 3016 - 3016

C. 939 - ☐ = 0

56.

57. Answer the following problems.

A. $\begin{array}{r} 16 \\ -\ 9 \\ \hline \end{array}$ B. $\begin{array}{r} 251 \\ -\ 6 \\ \hline \end{array}$ C. $\begin{array}{r} 92 \\ -\ 79 \\ \hline \end{array}$ D. $155 - 17 = \square$ E. $74 - 26 = \square$

59. Answer the following problems.

A. $\begin{array}{r} 872 \\ -91 \\ \hline \end{array}$ B. $\begin{array}{r} 1442 \\ -73 \\ \hline \end{array}$ C. $\begin{array}{r} 2222 \\ -383 \\ \hline \end{array}$ D. $401 - 308 = \square$

E. $723 - 156 = \square$ F. $\begin{array}{r} 43581 \\ -4709 \\ \hline \end{array}$

60. Problem: Paul wanted to make a rope for his boy scout pack. He was told that the rope cannot be longer than 25 feet. Paul decided to cut a 33 foot rope so that it is 24 feet long. How many feet of rope did Paul have to cut off the 33 foot rope?

APPENDIX C_2: Instructions for Giving the Semantic Differential Test at the Conclusion of the Study

The following test is a semantic differential attitude scale. You will find twelve sets of bi-polar descriptive adjectives in this test. Also, between the bi-polar adjectives are seven empty spaces. The children are to make a check in one of the seven spaces between each set of bi-polar items. The seven spaces are to allow for degree of feeling about a certain issue; in this case, feelings about their ability to do arithmetic. An example of an appropriate response would be:

hard ___ ___ X ___ ___ ___ ___ easy

The question to ask prior to the children filling out the attitude test is: "How does your ability to do arithmetic make you feel?"

If a child is not able to read or interpret the bi-polar items, make a note of it, and define the unknown words to him. If a child still has difficulty determining what you are asking him to do, then ask the following question for each set of bi-polar adjectives as you fill in the child's responses yourself: "How does your ability to do arithmetic make you feel: _____ or _____ or in between?"

APPENDIX C_2: Attitudinal Scale (Semantic Differential)

Big	___	___	___	___	___	___	___	Little
Unhelpful	___	___	___	___	___	___	___	Helpful
Old	___	___	___	___	___	___	___	Young
Strong	___	___	___	___	___	___	___	Weak
Powerless	___	___	___	___	___	___	___	Powerful
Deep	___	___	___	___	___	___	___	Shallow
Nice	___	___	___	___	___	___	___	Awful
Slow	___	___	___	___	___	___	___	Fast
Sour	___	___	___	___	___	___	___	Sweet
Alive	___	___	___	___	___	___	___	Dead
Good	___	___	___	___	___	___	___	Bad
Quiet	___	___	___	___	___	___	___	Noisy

APPENDIX C_3: An Introduction to KELP
(Kindergarten Evaluation of Learning Potential)

A Testing Instrument

KELP meets a need for an appraisal which enables each child to reveal his intellectual strengths early and in ways which are independent of any particular cultural orientation. The purpose of the Kindergarten Evaluation is to make available an instrument which samples individual behavior within a group of learners. The individual child's performance on KELP items, as part of the regular classroom activities, is observed and recorded by the teacher as an index of academic potential. This approach to observation sharpens and extends the skills of the kindergarten teacher and reinforces those patterns of interaction with the child which are most successful.

The child is observed over a period of months rather than for an hour or two. Three levels of learning are assessed:

LEVEL 1, ASSOCIATION: This is a process which includes operant conditioning. Copying on a step-by-step basis; trial and error or search and succeed; paired associate learning; and rote learning are examples of association learning.

LEVEL 2, CONCEPTUALIZATION: This second level of learning is built on the first level and is characterized by the grasping of whole ideas and by sudden insight. The child discovers the inherent structure, or grasps the relationships of the various parts, facts, or experiences in the learning situation.

LEVEL 3, CREATIVE SELF EXPRESSION: At this level, learning has elements of motivation and of unpredictability. It is creativity involving purposefulness or self-direction.

KELP Test Items

Following is a list of the KELP items and summarizations of the methods of evaluation.

1. SKIPPING: During instruction in rhythms and games, the teacher observes the ability of the child to skip on alternate feet.

2. COLOR IDENTIFICATION: Success requires the child to identify, by name, three primary and three secondary colors; and pink, brown, black and gray.

3. BEAD DESIGN: Success at Level 1 requires the child to copy five bead design cards, each having eight or nine beads and two or more repetitions

of the patterns. At Level 2 the child is asked to reproduce one of the designs from memory, and at Level 3 he creates a design of his own.

4. BOLT BOARD: This item consists of a wooden stand having nine holes of diminishing sizes fitted with bolts and nuts. Level 1 requires the child to take apart, mix, and assemble the bolt board. To attain Level 2 he explains the principles on which he worked. Level 3 requires that he show his own organization either in dismantling or in sorting the bolts for assembly.

5. BLOCK DESIGN: The child arranges nine colored blocks to match a printed block pattern; at Level 2 he does the same task using a miniaturized pattern with the outlines removed. At Level 3 he makes a design of his own.

6. CALENDAR: Discussions based on the typical kindergarten calendar provide a situation in which the child demonstrates, at Level 1, his ability to describe the date or the weather in a complete sentence. At Level 2 he demonstrates knowledge of the sequence of the days of the week. At Level 3 he is required to know the social significance of one of the holidays.

7. NUMBER BOARDS: These are plastic pieces of different lengths and colors having the top surfaces printed with units and with appropriate numerals. Level 1 requires the ability to count to ten and to recognize the numerals when they are presented in random order. Level 2 requires that the child demonstrate understanding of the inter-relations of any of the numbers from four to ten. At Level 3 he arranges and develops different groupings of eight, nine, or ten on his own initiative.

8. SAFETY SIGNS: Seven play-size signs are used to elicit responses basic to parts of reading readiness. At Level 1 the child reads or dramatizes correct recognition and copies five signs; at Level 2 he reproduces a sign from memory. At Level 3 he uses the letters available to make words of his own.

9. WRITING A NAME: At Level 1 the child learns to print his name in manuscript from the teacher's model; at Level 2 he prints his name with capital and lower case letters without a model. At Level 3 he prints words on his own initiative as needed to label or tell about his drawing or other work.

10. AUDITORY ATTENTION: This item consists of 18 small toys whose names begin with one of three consonants. At Level 1 the child identifies the objects, correctly articulating the names of the objects. At Level 2 he sorts the articles according to beginning sounds. At Level 3 he thinks of other words that begin the same way as the sounds taught.

11. SOCIAL INTERACTION: At Level 1 the child can report accurately what happened in a conflict situation. At Level 2 he can apply in a new situation any of the rules, agreements,or standards of behavior in the kindergarten. At Level 3 he acts on the rules he can express verbally.

(Items number 3, 4, 5, 7, 8, and 10 require the use of devices included in this kit. The other items to not require specially designed devices, but make use of the typical kindergarten setting.)

KELP Record Book

The KELP Record Book simplifies the task of recording observations of the child's performance. The (/) for "passed" is the only symbol that is counted for the prediction of later school success. Some other symbols (see Record Book) are very useful, if the teacher wishes to record them, in building a store of information as the basis for analysis of each child's growth pattern.

Separate squares are provided for each item for each month of the school year and for the three levels of learning. By marking (/) in the appropriate square, the teacher finds it possible to indicate the approximate time at which the child passed or attempted an item at a particular level, without having to write the date or any further notation.

The Record Book contains tables of norms necessary for converting the child's raw score to a stanine score. The stanine score is explained, including its relationship to other measures.

Summary Test Booklet

The Summary Test Booklet enables the teacher to administer an evaluation in the whole group setting and to compare the results of each child's performance against previous observations of highly individualized tasks. By having the children complete the booklet, the teacher is able to provide the first grade teacher with a current picture of each child's abilities.

The primary purposes of the Summary Test Booklet are: (1) to indicate what the child is able to do at the end of the kindergarten year by sampling his readiness abilities, (2) to provide a record of the child's earlier KELP successes on the KELP items for his cumulative file, and (3) to determine what the child remembers from his earlier KELP experiences.

Most tests of readiness for school work sample learning by association primarily. The KELP Summary Test Booklet includes conceptualization and creative abilities in addition to conventional readiness items. The booklet is designed in three parts, showing separate scores for association, conceptualization, and creative learning.

Directions for administering, and scoring the Summary Test Booklet, plus a pupil's booklet are features of the teacher's edition.

A Teaching Instrument

What the teacher learns to observe as levels of learning tends to become automatic and functions to enrich the total program. In association learning the problem is to bring the elements to be learned close enough together in space and time so that they can be grasped by the child during a period of attention. Some examples of kindergarten activities that involve association learnings include memorizing the prices of groceries in the play store, knowing the location of work materials in the kindergarten room, and learning the words and tune of a new song.

The teacher who has become accustomed to moving to other levels of learning as work with KELP progresses, is able to encourage Level 2 functioning in areas where this development of conceptualization has not been attempted as a regular practice. A discussion which follows the reading of a story can be changed from a review of the factual content of the story to a discussion of significant meanings. To stress the motives of the characters, to identify the values expressed in the story, or to ask the listeners to predict the next event are examples of teaching for Level 2 thinking. Kindergarten teachers, as a result of training and experience, use many techniques which stimulate children's conceptual development, but KELP provides clear guidelines to observe when conceptualization occurs in a learner. In addition, KELP helps the teacher identify the conditions which increase the Level 2 thinking in the class.

In much the same way the teacher who uses the KELP items to encourage creativity on the part of the children is encouraged to plan additional opportunities for them to practice independent action. Encouraging children to accept leadership roles in play and to initiate playground activities are sometimes overlooked by teachers and parents as ways of reinforcing creative behavior. The bonus value for the teacher is to see how the practice of leadership and self-direction changes those children who might not have developed these traits on their own.

Guidelines for teaching each item are suggested in the professional book, Kindergarten Evaluation of Learning Potential, by John A.R. Wilson and Mildred C. Robeck. The following guidelines for Auditory Perception are cited as examples.

The speech of most young children reflects both nature and nurture, the latter being in part the responsibility of the school. Much of the work which the kindergarten teacher does is to increase the intent and the precision of the child's vocabulary through stories, poems, sharing experiences, and similar activities. One purpose of teaching the auditory perception item is to extend vocabulary by naming the objects and in addition, to provide a meaningful structure in which the child's vocabulary development can be observed.

One effective way to introduce this item is to bring the toys from any one compartment to the sharing period. The toys are shown and different children are given a chance to name them. The teacher may then ask whether anyone can name all of the toys. As the toys are named, children begin to associate the appropriate words with the toys. Hearing a sequence of names that begin alike, the children get auditory stimulation through the sound repetitions -- the first step in auditory training for phonics.

For children who do not pronounce correctly, the teacher provides a model by her own correct enunciation of the word.

On other days the toys from each of the compartments are introduced. By this time the teacher will have observed which children are likely to be successful in naming the objects. One or more of the children may be observed at once for the ability to score at Level 1.

During this process the most perceptive children may have recognized the common sound in one or more of the groups. Occasionally the teacher will need to suggest that they listen to these names. Is there anything special about them? The selection of objects having names of one, two, or more syllables facilitates discovery that the word likeness occurs at the beginning of the word.

At this point, the teacher may introduce the use of the key words on the Auditory Perception box as a guide for sorting at Level 2. In the typical kindergarten, one or more children will be able to sort the toys by beginning sounds as soon as the three groups of objects have been presented, and occasionally a child will be able to sort the toys by beginning sound before he articulates clearly enough to score at Level 1.

At Level 3, the children can name visible objects around the room that begin like any one of the key words. Some children may mention words which they have encountered in play and home experiences. Sometimes children, while involved in a completely different activity, discover that a word begins like another word. For example, while the group is saying a favorite nursery rhyme or singing a song, a child may suddenly say that a word from the song or rhyme begins like turtle. The teacher can encourage these discoveries by recognizing and acknowledging the child's contribution. At this point the teacher may even pose a question which directs the children to listen for other words in the poem which begin like turtle.

In general, the following principles are basic to teaching all the items in KELP: The teacher incorporates the KELP material into the regular curriculum. KELP items are taught as thoroughly as the children are able to learn them at the time.

Procedures are planned so that different children learn at varying levels. The teacher functions according to her own professional personality. The teacher's skill as an observer is used to advantage to accomplish the appraisal.

APPENDIX D

D_1: Case Study Background Information Form

D_2: Instructional Activities Form

APPENDIX D_1: Case-Study Background Information Form

I. Identification Data (to be coded)

Full name and Nickname

_____
(last) (first) (middle) (nickname)

Hypothetical Name_____

Address:_____
(city and state)

Age_____ Sex_____ Birthdate_____
(months/days/years)

Name of school district where home is located_____

_____

Name of homeroom teacher_____

Name of teacher in the study_____

Date of the report_____

II. Test Data

Identification of test taken:

_____
(name of test) (form used) (date administered)

_____

(results of test)

_____

_____

_____

_____

_____

_____

Researcher-Designed Diagnostic-Achievement Test

Performance at beginning of study

Addition _____

_____

_____

Subtraction _____

_____

_____

Performance at the conclusion of study

Addition _____

_____

Subtraction _____

_____

III. Educational History

Age child entered school _____

Preschool training _____
(kindergarten, nursery school, etc.)

Number of grades child has repeated _____ Which one(s) _____

Number of schools child has attended _____

Approximate number of teachers child has had _____

Mathematics achievement report for each year in school:

Grade 1: _____

Grade 2: _____

Grade 3: _____

Grade 4: _____

Grade 5: _____

IV. General description of physical conditions -- include grade level at which it occurred

_____

_____

_____

V. General description of environmental background -- include dates of specific happenings

_____

_____

_____

VI. Interests and attitudes

_____

_____

_____

VII. Diagnosis

	date mastered	not mastered
Addition:		
Knowledge of concrete materials	_____	_____
Discriminate and classify differences not in a set of objects	_____	_____
Discriminate and classify characteristics logical to objects	_____	_____
Recognition of sets	_____	_____
One-to-one correspondence	_____	_____
Equivalence	_____	_____
Greater than or less than	_____	_____
Meaningful counting	_____	_____
Empty set	_____	_____
Write meaningful symbols for 0-9	_____	_____
Cardinality	_____	_____
Set union	_____	_____
Commutative property of union	_____	_____
Associative property of union	_____	_____
Principle of addition	_____	_____
Identity element of addition	_____	_____
Addition facts with sums less than ten	_____	_____
Regrouping ones to tens	_____	_____
Addition facts with sums greater than nine but less than 18	_____	_____

	date mastered	not mastered
Commutative property of addition		
Associative property of addition		
Place value & regrouping		
Compute by regrouping & non-regrouping		
Setting up & solving word problems involving addition		
Ability to read problem		
Identify question		
Identify operation of addition		
Identify relevant data		
Write number sentence for addition problem		
Set up vertical addition algorithm		
Principle of subtraction: Addition & subtraction as inverse processes		
Subtraction facts, $a - b = c$, $a \leq 9$		
Commutative & associative properties of addition		
Subtraction facts with minuend greater than nine		
Subtracting zero		
Subtracting a number from itself		
Compute by regrouping tens to ones		
Compute by regrouping (borrowing) and non-regrouping		
Setting up & solving word problems involving subtraction		
Ability to read problem		
Identify question		
Identify operation of subtraction		
Identify relevant data		
Write number sentence for subtraction problem		
Set up vertical subtraction algorithm		
Principle of multiplication: Multiplication facts		
Multiplication by zero		
Multiplication by one		
Commutative property of multiplication		

	date mastered	not mastered
Associative property of multiplication		
Distributive property of multiplication		

VIII. Instructional Procedures

Sequential lists of skills which the child mastered during the study

Instructional activities and exercises which enabled the child to master each skill

Teacher's evaluation of each activity

IX. Summary and Recommendations

Analyze and bring together information found in the form of an instructional package for each subject studied.

APPENDIX D_2: Instructional Activities

Date_____

Skills being taught_____

Behavioral Objective (briefly stated):_____

_____

_____

Description of Instructional Procedures: (indicate what you did, child did, visual aids used, etc.)

_____

_____

_____

_____

Examples of exercises child performed for this skill: (give examples of the manipulations or verbal problems and attach worksheets)

_____

_____

Degree of Success: (your evaluation of the activity; did the child meet the criteria specified in the behavioral objectives; how could it be improved; why was it successful or not successful?)

_____

_____

_____

_____

Homeroom activity described briefly on back: (please staple worksheets, etc. to this paper)